Calling Plans Lesson – Part 1 Algebra

Overview: In this lesson students compare two linear relationships in the context of phone calling plans. Students are required to construct and interpret a table, a graph, and an equation for each calling plan to help them decide when one calling plan is a better deal than the other. This lesson is designed to focus on the standards for Quarter 1 while at the same time building the foundation for standards in subsequent quarters. The lesson could be extended so as to specifically address Standard 9.0 in Quarter 2. Student work for this problem should be kept so that it can be used in subsequent quarters when discussing the problem. The following notation is being used to differentiate between quarters:

❶ Quarter 1
❷ Quarter 2

Goals:

❶ • Students will solve the problem using an equation, a graph, and a table.
❶ • Students will interpret a linear equation in terms of the problem.
❶ • Students will justify their solutions to the problem.
❶ • Students will construct and interpret the graph of a linear relationship in terms of the problem.
❶ • Students will identify and interpret the x- and y-intercepts of the graph of a linear relationship in terms of the problem.

❶ **Algebra Standards:**

❶ • 5.0 Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and *provide justification of each step*.
❶ • 6.0 Students graph a linear equation and *compute* the x- and y-intercepts.
❶ • 7.0 Students *verify* that a point lies on a line, given the equation of the line. (Quarter1) Students are *able to derive* the equation of a line using the point-slope formula. (Quarter 2)
❷ 9.0 Students *solve* a system of two linear equations in two variables algebraically and are *able to interpret* the answer graphically.
❷ 24.0-25.0 The reasoning standards are in italics within the algebra standards. When mathematical reasoning is expected in the lesson, the text will be labeled Mathematical Reasoning within the text.

Building on Prior Knowledge: Seventh grade Standards

3.0 Students graph and interpret linear and some nonlinear functions.
4.0 Students solve simple linear equations and inequalities over the rational numbers.

Materials: Calling Plans task (attached); graph paper; calculators; rulers; chart paper; markers.

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	Action	Comments
S E T U P	PRIOR TO THE LESSON • arrange the desks so that students are in groups of 4 • determine student groups prior to the lesson so that students who complement each other's skills and knowledge are working together • place materials for the task at each grouping • solve the task yourself	PRIOR TO THE LESSON It is critical that you solve the problem in as many ways as possible in preparation for this lesson so that you become familiar with strategies students may use. This will allow you to better understand and respond to students' thinking. As you read through this lessons plan, different strategies for solving the problem will be discussed
	HOW DO I SET-UP THE LESSON? Ask: **Does anyone have a cell phone? What is important to keep in mind when you purchase a cell phone?** Get students to bring out ideas such as monthly cost and cost per minute. Ask students to follow along as you read the problem. Then have several students explain to the class what they are trying to find when solving the problem. Tell students that they are expected to make a table, write an equation, and draw a graph. Their solutions should be displayed on a poster that contains all 3 representations. Also, stress that students will be expected to explain how and why they solved the problem in a particular way and what the solution means in context of the problem.	HOW DO I SET-UP THE LESSON? As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate the goal is to find out when Company A's plan becomes cheaper than Company B's plan. It may be helpful to have students identify that the problem involves cost (money) and time in minutes. Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving. Students will be more successful in this task if they understand what is expected in terms of group work and the final product.
E X P L O R E	INDEPENDENT PROBLEM-SOLVING TIME Give students 5 - 7 minutes of private think time to begin to solve the problem individually.	INDEPENDENT PROBLEM SOLVING TIME Make sure that students' thinking is not interrupted by the talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes.

SMALL GROUP RPOBLEM SOLVING

Have students continue to work on the problem in their groups. Circulate among the groups assessing students' understanding of the idea below.

What do I do if students have difficulty getting started?

Assist students/groups who are struggling to get started by prompting with questions such as:

- What do you know about the phone plans?
- What would be the cost for talking 1 minute in Company A's plan? 2 minutes? 5 minutes?
- What would be the cost for talking 1 minute in Company B's plan? 2 minutes? 5 minutes? Can you find a way to determine the cost for any number of minutes?

What misconceptions might students have?

Look for and clarify any misconceptions students may have.

a. *incorrectly writing 4 cents as .4 rather than .04.* **How would you write 40 cents? OR Write down $.40 and $.04 and have students explain the difference between the two amounts.**

b. *mixing dollars and cents notation.* (e.g., 5 represents $5 and 4 represents 4 cents. This would lead to an incorrect equation of C = 5 + 4m.) **What does the 5 represent? What does the 4 represent? How would you write each in dollar notation? In cents notation?**

c. *misinterpreting the monthly fee and the per minute fee (e.g., You will see C =.04 + $5m instead of C $5 + .04m).* **If we are talking about the cost for one month then what does the $5.00 and the $.04 mean?**

SMALL GROUP PROBLEM SOLVING

What do I do if students have difficulty getting started?

By asking a question such as "What do you know about the phone plans?" the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine.

By beginning with one minute, students can begin to see what changes and what stays the same. In this case, that the fix fee ($5 and $2) stays the same and is added onto the charge for the number of minutes they used the phone.

What misconceptions might students have?

Misconceptions are common. Students may have remembered the information incorrectly or they may generalize ideas prematurely. Some strategies for helping students discover when they have made an error include:

- Create comparison situations (e.g., $.04 and $.40)
- Press students for the meaning of the numbers within the context of the problem. Consistently asking about the context helps students to make sense of the problem.

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?
Students will approach the problem using a variety of strategies. Some strategies are shown below. Questions for assessing understanding and advancing student learning are listed for each.

using a table
Students' tables might have the following characteristics:
- Showing the number of minutes talked with the cost of both plans side-by-side, or each plan in a separate table.
- Incrementing the number of minutes by 1, by 5's and by 10's.
- Excluding the base fee from the table (cost for 0 minutes).
- Creating a table for only 1 plan and failing to show a comparison between the costs of the two plans.

- *Monthly Fee Not Shown On the Table*
If students have not included the monthly fee in constructing their table (i.e. did not show that at 0 minutes Plan A would cost $5 and Plan B would cost $2), ask: **According to your table one minute cost $5.04. How can one minute cost $5.04 when one minute cost $.04? Where does the $5.00 come from?**

- *Student works with increments of 1 minute*
If students begin constructing their table in increments of 1 minute, ask or say:
- **Plan A begins with $5.00 and Plan B begins with $2.00. Plan A goes up by .04 each minute and Plan B increases by .10 each minute. What do you think will happen if they keep growing this way? It's going to take a long time for us to see when Plan A becomes the better deal if we keep going by 1 minute at a time. What else could you do so it won't take so long?**

- *Student believes that Plan A is the better deal at 50 minutes*
If students think they reached the solution of 50 minutes, ask:
- **Which is the better deal: Plan A or Plan B? How do you know?**
- **Is Plan A a better deal at 50 minutes? How can you tell?**

- *Student claims that Plan A is the better deal at 55 or 60 minutes.*
Play Devil's Advocate: **Others students think that Plan A is the better deal at 51 minutes. You think it is the better plan at 55 minutes. Which one do you think is right? How do you know?**

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?
The teacher's role when students are working in small groups is to circulate and attend to what students are doing and saying. The teacher should ask questions as needed in order to determine what students understand about the problem (and the mathematics) and to advance students' understanding of mathematics.

using a table
Most students will be able to create a table that includes time and cost. Students may, however, have difficulty determining an appropriate interval and figuring out the answer from the table.

- *Monthly Fee Not Shown On the Table*
It is important to prompt students to include the monthly fee in their table because it will help them understand the role of a base fee in the equation and to locate the y-intercept on the graph.

Mathematical Reasoning: Students should explain that the companies charge $5.00 or $2.00 even when 0 minutes have been used.

	using an equation If students have not written an equation prompt them to do so by asking: **- What do you know about the cost of using each plan?** **- How would you figure out the cost of Company A for 100 minutes? 200 minutes? Any number of minutes?** **- You said that the cost per minute is $.04. How can you write this to show $.04 per-minute no mater how many minutes we talk?** **- If we talk 1 minute at .04, 2 minutes at .04, 5 minutes at .04, or a hundred minutes at .04? If you add on the base/initial fee, then how would you write this in the number sentence?** *Work with groups of students so equations appear on charts for the group discussion.*	using an equation Students should be able to state that the cost per minute must be multiplied by the number of minutes and that the monthly fee must be added on. Asking students to describe the plan, then formalizing their verbal description of the plans will help students make connections between their informal problem-solving strategies and formal symbolic equations. If students are having difficulty writing an equation refer to the information in the table. As students point to the cost for the plan, ask them how they arrived at the cost. When they tell you that they multiplied by the number of minutes by .04 then tell them to write this down. Ask the student what amount this gives them. The student will notice that the base not been included. Ask them how you will add it on.

	using a graph - *Independent and Dependent Variable* To assess student understanding of the dependent and independent variable ask: **- What goes on the x-axis and what goes on the y-axis? Why?** **- Does the time you talked on the phone depend on the cost or does the cost of the plan depends on how long you talked?** - *Scale of the Graph* **- What units are we dealing with for each axis?** - Tell students: **The scale for the x- and y-axis does not have to increase by the same increment, but each axis must increase by a fixed increment.** - *The Point of Intersection and the Solution to the Problem* Ask students to indicate where the solution to the problem is on the graph. If they indicate that it is the point of intersection, ask: **What does the point of intersection mean in this problem?**	using a graph - *Independent and Dependent Variable* Students should label the x-axis as their "time" or "minutes" axis and the y-axis as their "cost" axis. Mathematical Reasoning: Students should explain that the cost depends on the number of minutes used. - *Scale of the Graph* Students should identify x axis as minutes and y axis as cost Look for students who do not use fixed increments (e.g. some will start with 1 min., 2 min., and then jump to 10 min. and 20 min.) Students may believe they must use the same scale for both axes. If they attempt to use the same scale, the x (minutes) axis must go beyond 50 while the y (cost) axis only needs to go past 7. Therefore, it will be difficult to physically construct the graph. - *The Point of Intersection and the Solution to the Problem* Mathematical Reasoning: Students should state that the point of intersection is the point where both plans cost the same for the same number of minutes. It is important to hear students talk about both the cost and minutes. They also need to state that the point of intersection is not the solution. Rather, the solution is that after that point, Plan A is cheaper than Plan B beyond the point of intersection. They should be able to explain, using the graph, that after the lines intersect the graph of Plan A is below the graph of Plan B.

S **H** **A** **R** **E,** **D** **I** **S** **C** **U** **S** **S,** **A** **N** **D** **A** **N** **A** **L** **Y** **Z** **E**	GROUP DISCUSSION *In what order will I have students post solution paths so I will be able to help students make connections between the solution paths?* As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of the different parts of the table in terms of the problem and the different parts of the graph in terms of the problem. Ask students to post their work in the front of the classroom. The goal is to discuss mathematical ideas associated with cost-per-minute (rate of change/slope) and monthly fee (y-intercept) for each plan. Students should be able to say why one plan starts out as the less expensive plan but over time it is the most expensive plan. They should understand how this information is represented in a table, a graph and an equation.	GROUP DISCUSSION *In what order will I have students post solution paths so I will be able to help students make connections between the solution paths?* Even though you may display the work produced by all groups, you should strategically pick specific examples to discuss with the whole group. You might want to choose solutions in which tables show different time increments as well as making certain you choose a solution that shows the correct graph. It would also be helpful to have a solution with a correct equation so that connections can be made among the 3 strategies. Analyzing incorrect solutions can also be a way of helping students develop the ability to detect and correct errors and to explain their thinking about what does and does not work and why. Students might say that one plan starts out more because one plan has a base fee of $5.00 and the other has a base fee of $2.00. Over time, the plan with a $5.00 base fee becomes less expensive because the cost-per-minute for this plan is less than the cost-per-minute of the other plan. Students should make a comparison between the base fees and the cost-per-minute of each plan when justifying their response.

*What questions can I ask during the discussion that will serve to highlight the mathematical ideas (slope/rate of change, y-intercept, etc.) and the connections between different ways of representing the ideas (table, graph, equation)? (Driving Questions *)* **When does Plan A become the better plan? How do you know from the table?** (Direct students' attention to a table that shows the two plans.) Some students will say that Plan A is the better plan at:

- 50 minutes
- 55 minutes
- 60 minutes

Ask students: **Can you show me in the table or the graph how you know when Plan A is the better plan?**

a. Interpreting the Information in the Table
Ask students: **Where do you see the solution in the table?** Some students will say: - 50 minutes,
- some will say 51 minutes,
- some will say 60 minutes
- some will say 55 minutes

Press students to take a position on WHEN Plan A becomes cheaper and to support their claims with evidence from the graph or table. This usually leads to a discussion about the cost-per-minute.

Students will share the solutions for 50, 51, 55, and 60 minutes and should realize that 51 minutes is when Plan A becomes the better deal. This is a good time to talk about the point of intersection. You can ask: **Where is the solution shown on the *graph?*** Some will refer to the point of intersection and others will refer to the point after the intersection. This creates another opportunity for further discussion by your asking:

Point of Intersection
- **When does the graph show Plan A becoming the better deal?**
- **What does this point represent with respect to the calling plans?**
Listen for reference to the cost and the number of minutes (e.g. $7.00 at 50 minutes.)
- **How would you write the point of intersection as an ordered pair?**
Students should indicate that it would be (50,7) and be able to explain the relationship between the ordered pair and the point on the graph.
- **Where do you see the point of intersection in the table?**

What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? Driving questions (*) Driving Questions have been provided that will help to stimulate student interest and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or problem solving strategies.

Accountable Talk
Pointing to Information in tables and Graphs
Asking students to point to places in the table or graph will help others in the class follow the conversation.

Repeating or Paraphrasing Ideas
Ask other students to put explanations given by their peers into their own words. This is a means of assessing understanding and providing others in the class with a second opportunity to hear the explanation.

Position-Driven Discussion
Press students to take a position and to support their claims with evidence from the graph or table. Students must say why they believe Plan A becomes a better plan at a given number of minutes. In doing so they will have to provide reasons for their claims.

Tables, Graphs, and Equations for the Calling Plans appear below.

Referring to a Table

Time (in min.)	Cost (Plan A)	Cost (Plan B)	
0	$5.00	$2.00	Monthly fee (cost at 0 minutes;y-intercept)
10	$5.40	$3.00	
20	$5.60	$4.00	
30	$6.40	$5.00	
40	$6.60	$6.00	
* 50	$7.00	$7.00	Same cost (point of intersection)
51	$7.04	$7.10	Plan A is less expensive than Plan B
55	$7.20	$7.50	
60	$7.40	$8.00	

Cost Per Minute

- Why did some of us think that Plan A became the better deal at 60 minutes and others think it was 55 minutes?

- How do we see the cost-per-minute for each plan in the *table*? Where is the $.04 in the table? Where is the $.10 in the table?

- Follow the same line of questions for Company B.

- How would you write an *equation* that will help you figure out the cost for 32 minutes if you use Company A? Company B? Why did you multiply 32 x .04? Why do we add $5.00 or $2.00?

Once students have written an equation, give students a minute to solve for the cost of 32 minutes in each plan. Have students verify that these points (32, 6.28) and (32, 5.20) are in fact points on the lines by approximating where they would be using the graph.

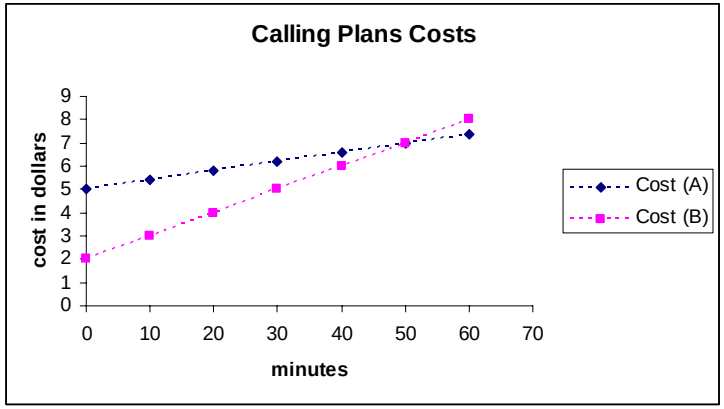
Driving Question

*** How can Plan A end up being the better plan when it starts out costing the most and Plan B starts out costing the least?**

Referring to an equation

Company A	Company B
C = .04m + 5.00	C = .10m + $2.00

Referring to a graph



Cost Per Minute

It is worth discussing the fact that the cost per minute is within each table but "hidden" when using increments greater than 1 minute. If students are struggling with this concept, choose 2 consecutive entries for Company A (15 minutes -$5.60 and 20 minutes - $5.80) and ask students to talk about the cost per minute. In this case, students should see that 5 minutes cost an additional $.20 and that this is the same as $.04 for one minute. Do the same for Company B.

Mathematical Reasoning: *The coefficient* - When you ask students why they multiplied 100 (.04) or 100 (.10) they should say that every minute that you talk on the phone you are charged .04 or .10 minutes so if you talk 100 minutes this is 100 times the .04 or .10. *The constant* - Students should explain that the initial fee is added on because it is a one-time fee, not a per-minute fee.

y-Intercept or Monthly Fee

- Point to 0 minutes in the *table* and ask: **What does this mean in terms of the phone plans?**
- Point to the *graph* and ask: **One plan begins at $5.00 and the other at $2.00. What does that mean with respect to each plan?**
- **Is the monthly fee shown in the *equations*? Why did they add the fee on?**
- Mark this information by saying: **We call the $5.00 and $2.00 the y-intercepts because this is where the number of minutes (the x value) is equal to zero and where the *lines* touch or cross the y-axis.**
- **Will these lines ever cross the x-axis?**

Driving Question

*** Plan A has an initial fee that is greater than Plan Bs initial fee, so how can Plan A end up saving you money?**

Slope/Rate of Change

Prompt a discussion with the following question:

- **What makes Company A end up being a better deal than Company B?**
- Emphasize the way the plans are growing by saying: **The rate of growth for each plan is $.04 and $.10 per minute. What does this have to do with the steepness of lines?**
- Press students further by asking: **Plan A charges more for the base fee and less per minute. Help me understand how we end up making up for the difference between the two plans initial fee?**
- Invite students to use a yardstick and show the graph for each of the lines represented by the equations below:

C = $5.00 + .06x
C = $5.00 + .08x
C = $3.00 + .04x

y-Intercept or Monthly Fee

Students should state that the y-intercepts are the costs of the plans for talking 0 minutes. They may call the amount the base fee or the service charge.

Mathematical Reasoning (x-axis): When asked if the graphs will ever cross the x-axis, students should say that the line will cross the x-axis when x is negative. But that, in this context, you can't have negative minutes so we stop at x = 0. Show the line extending on the graph so students can explain the reason why the line must be limited to the positive values for x (minutes) and y (cost).

Using an equation

Company A	Company B
C = .04(0) + 5	C = .10(0) + 2
C = 0 + 5	C = 0 + 2
C = 5	C = 2

Company A charges $5 for 0 minutes and Company B charges $2 for 0 minutes.

Slope/Rate of Change

Mathematical Reasoning (Slope): Students will say Plan B goes up by .10 and Plan A goes up by .04. Refer to the going up at the same rate as *the rate of change*. Talk about the slope as the rate of change per minute. For each additional minute talked the plan rises or increases by $.04 or $.10 minutes. The relationship between the minutes and the cost should be explicit.

Mathematical Reasoning: Students should say that Company B makes up the cost charged initially by Company A after 50 minutes. Since per-minute-cost is greater for Company A than it is for Company B, it will take 50 minutes for Company B to recover the $3.00 difference between the two companies' initial fees. Every minute used on Company B's plan earns $.06 more than Company A's plan. After 50 minutes the $3.00 difference between the two companies' initial fees will be recovered.

Mathematical Reasoning (Extension Expressions): Students should be able to say that $3.00 + $.04 is parallel to Company A's plan because the per-minute-cost is the same. (It has the same steepness as Plan A.) Students should explain that $5.00 + .06x and $5.00 + .08x will be steeper lines than Company A's line.

	HOMEWORK You could give the following problems as a homework assignment: Company C has no monthly fee but charges 12 cents a minute. Using a table, a graph, and an equation, determine when Company A would be cheaper than Company C. OR Create phone plans that would be cheaper than company A if: a. you didn't use the phone very often. b. you used the phone all of the time.	

Calling Plans

Long-distance Company A charges a base rate of $5 per month, plus 4 cents per minute that you are on the phone. Long-distance Company B charges a base rate of only $2 per month, but they charge you 10 cents per minute used.

How much time per month would you have to talk on the phone before subscribing to Company A would save you money?

(Achieve, Inc., 2002)