

The Candy Jar Task: A Ratio and Proportion Lesson

Overview: Proportionality is an important integrative thread that connects many of the mathematics topics studied in grades 6–8. Students in grades 6 – 8 encounter proportionality when they study linear functions of the form $y = mx$, when they use the relationship between the circumference of a circle and its diameter, and when they reason about data from a relative-frequency histogram. In this lesson students will use ratios to show relative sizes of two quantities and will be asked to understand and use ratios and proportions to represent quantitative relationships by using information about the number of Jolly Ranchers and Jawbreakers in a jar. This lesson is designed to focus on 6th grade standards for quarter 2.

Goals:

- Students will understand relationships in which two quantities vary together and the variation of one coincides with variation of the other
- Students will understand the multiplicative nature of proportional reasoning
- Students will develop a wide variety of strategies for solving proportion and ratio problems

6th Grade Quarter 2 standards:

- NS 1.2 Interpret and use ratios in different contexts to show the relative sizes of two quantities, using appropriate notations (a/b , a to b , $a : b$).
- NS 1.3 Use proportions to solve problems. Use cross multiplication as a method for solving problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.
- MR 1.1 Apply strategies and results from simple problems to more complex problems.
- MR 2.7 Make precise calculations and check the validity of the results from the context of the problem.
- MR 3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

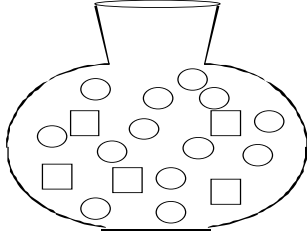
Materials: Ratio and Proportion Tasks; Two colors or shapes of counters, calculators; chart paper; markers

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Phase	Action	Comments
S E T U P	1) Prior to the Lesson: a. arrange the desks so that students are in groups of 4. b. Determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. c. Place materials for the task at each group. d. Solve the task yourself. * This lesson contains three tasks. The first task includes an Explore as well as a Share, Discuss and Analyze Component. The second two tasks share the Explore and Share, Discuss and Analyze Component. <u>HOW DO I SET-UP THIS LESSON?</u> Allow students time to explore the meaning of ratio by introducing ratio as a way of comparing two quantities. As you sketch at the overhead, ask students to also sketch at their desk the following: • 2 “stick figures” for 4 lollipops. • 4 “stick figures” for 8 lollipops. • 6 “stick figures” for 12 lollipops. Ask the following questions: • How many lollipops does one person receive if there are four lollipops for every two people? How do you know that each person will receive two lollipops? • If there are eight lollipops for every group of four people then how many lollipops will one person receive? How do you know that each person will receive two lollipops? • If six people receive twelve lollipops then how many will	1) Students will be more successful if they understand what is expected in terms of group work and the final product. After you share the task with students you can ask them what they should include in their work to make it quality work. They will say: - neat work. - a written explanation - a problem and correct answer If students do not say a picture or table you might suggest that these can be used because they might help others understand how they solved the problem. d. It is critical that you solve the problems in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different strategies for solving the problems will be given. <u>HOW DO I SET-UP THIS LESSON?</u> As students sketch and decide on ratios equivalent to the lollipops: “stick figures” circulate and listen to students understanding of the multiplicative relationships between equivalent ratios. It is important to NOT tell students how to find equivalent ratios, as it is your goal to increase conceptual understanding and flexibility with ratios. The questions have been stated carefully to highlight the proportional relationship. (2:4 as 1 is to what, 8:4 then what is to 1). Keeping the relationship consistent is important when asking the question. Record the proportions formally as students share the relationship. e.g. 2:4 as 1:2

	one person receive? What do you notice? What can we say about these problems?	4:8 as 1:2 if students talk about the lollipops then record as 8:4 as 2:1 (Ask students if the two ways of recording and thinking about the lollipops mean the same thing.) Possible Responses - I thought about 8 and split them up between the 4 people. <i>Revoice the students' contribution, so he hears you say, "You divided 8 by 4 and you knew each person would receive two. So eight is to four in the same way as two is to one."</i> - I knew that 1 person would receive two because one person gets two, then two people get four, three people get six, four get eight, five get ten, and six get 12. <i>Revoice as "So you knew that there were two for every person and since there are six people you said $6 \times 2 = 12$." This revoking of a student's contribution highlights multiplicative thinking rather than additive thinking.</i> What should students notice? Students might be able to say: - Even though we are talking about 2 people and twelve lollipops we are still talking about two lollipops for every person. - You can say 6 is to 12 in the same way as 1 is to 2 or you can say 12 is to 6 in the same way as 2 is to 1. You can go backward as long as the relationship remains the same.
S E T U P	Extension Activity Other possible ratios for discussion are: 3 apples to 6 students; 6 students to 18 mini pizzas 24 pieces of gum to 4 treat bags	These experiences prepare students for the next task that will provide students with additional opportunities to gain an understanding that ratios are expressed as a mathematical relationship involving multiplication.

Give students the attached Jolly Rancher and Jawbreaker task.



- a) What is the ratio of Jolly Ranchers to Jawbreakers in the candy jar?
- b) ~~Write, as many ratios as you can that are equivalent to the first ratio that you wrote down,~~

PRIVATE PROBLEM SOLVING TIME

Give students 3 – 5 minutes of Private Time to begin to individually answer the problems.

FACILITATING SMALL GROUP PROBLEM SOLVING

What do I do if students have difficulty getting started?

Now ask students to work in their groups to solve the problems. Assist students/groups who are struggling to get started by prompting with questions such as:

- How do the Jawbreakers compare with the Jolly Ranchers?
- How is this problem similar to the Lollipop problem? What were some equivalent ratios in that problem? Can you think about this problem in the same way as you thought about the Lollipop problem?
- Remember when we said that 4 people were to 8 Lollipops in the same way as 1 person is to 2 lollipops? Can you think about the same kinds of relationships between the Jolly Ranchers and the Jawbreakers?
- What does it mean to say two ratios are equivalent?

Student experiences with the set-up questions prepare them for question a).

Possible Responses to Question a.

5Jr: 13Jb or 13Jr: 5Jb

Possible Responses to Question b.

1 Jr : 2.6 Jbs

5 Jrs : 13 Jbs

10 Jrs : 26 Jbs

15 Jrs : 39 Jbs

20 Jrs.: 52 Jbs

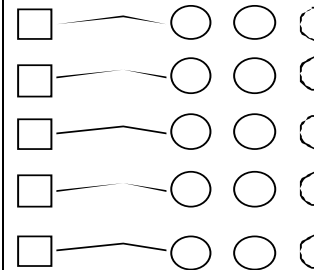
PRIVATE PROBLEM SOLVING TIME

Make sure that student's thinking is not interrupted by the talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes.

FACILITATING SMALL GROUP PROBLEM SOLVING

What do I do if students have difficulty getting started?

If students are experiencing difficulty then ask them to show you the relationship with the counters.



- If 5 JB is to 13 JR then what do you know about one JB? How many JR are related to one JB?
- What numbers other than 5:13 could you use to describe the relationships between Jawbreakers and Jolly Ranchers?
- What if there were 26 Jbs then how many Jrs would there be?

Below are examples of questions that can be used throughout this portion of the lesson.

- What do you know so far about ratios?
- How do you know your ratios are equivalent?

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What misconceptions might students have?

- Press students for the meaning of the numbers within the context of the problem, e.g., What does it mean to have a 5:13 ratio? What does the 5 mean? What does the 13 mean?
- Ask students how they know if the same relationship exists between two ratios. Ask them how they know they are equivalent.
- How can you be sure that the 10:26 ratio is in proportion to the 5:13 ratio?

What strategies might students use?

Students might make a table of ratios

5 jrs	13 jbs
10 jrs	26 jbs
15 Jrs	39 Jbs
.	.
.	.

- What did you learn earlier that would help you with this problem?

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What misconceptions might students have?

Misconceptions are common. Students may have learned information incorrectly or may have generalized ideas prematurely. Some strategies for helping students discover they have made an error include:

- Students may be uncertain about how to write ratios; $\frac{3}{4}$, 3 to 4, 3 : 4
- Students may not distinguish between proportional situations and additive situations. e.g, If you ask students how many Jolly Ranchers there are if you have 26 Jawbreakers, then some students will add 13 Jolly Ranchers to 5 since they added 13 Jawbreakers to 13 to get to 26 Jawbreakers.

What strategies might students use?

The table presents an opportunity to explore unit- rates and relate that discussion to the multiplicative properties of proportional thinking during the discussion portion of this lesson.

	Student might circle 5 Jolly Ranchers to every 13 Jawbreakers, 5 more Jolly Ranchers to 13 more Jawbreakers, etc. as a way to think about the relationship between the two quantities.	Although this focuses on the relationships of the 5:13 ratios, it may indicate a beginning understanding of proportional thinking.
S H A R E, D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE GROUP DISCUSSION</u> <i>What order will I have student share their solutions so I will be able to help them make connections between ratio and proportions?</i> As you circulate among groups of students, listen for how they are thinking about ratio and proportion. Mentally identify the students whose thinking you want the entire class to hear and in what order you want them to hear it. If there is a student with a table based on additive reasoning and you know the student feels safe to be wrong, begin the class discussion with that student's work. By choosing this incorrect strategy first, a discussion about additive vs. multiplicative thinking can take place. Next, because of the mathematical importance of tables, ask another student with a table based on correct multiplicative thinking to share the table with the class. Be sure to focus on the multiplicative attributes of the table.	<u>FACILITATING THE GROUP DISCUSSION</u> <i>What order will I have student share their solutions so I will be able to help them make connections between ratio and proportions?</i> As students share their solutions with the class, make sure the focus of the discussion is on the mathematics of ratio and proportion. Your reason for student sharing is to increase all students understanding of the meaning of ratio and proportion. It is critical that you emphasize conceptual understanding of proportional thinking rather than procedural competence.

	<i>What questions can I ask during the discussion that will help students keep the context and the goal of the problem in mind?</i> Ask: - If I keep the same 5:13 ratio, how many Jawbreakers will I have if I have 1 Jolly Rancher? Does the same relationship between Jawbreakers and Jolly Ranchers still exist? How do you know? - What if I tell you that there are 10 Jolly Ranchers then how many Jawbreakers are there? How do you know? Does the same relationship still exist between the Jolly ranchers and the Jawbreakers? What does this mean? - What if I tell you that there are a lot of Jawbreakers, that there are 52 then how many Jolly Ranchers would there be? How did you figure it out? Does the same relationship still exist between the Jolly Ranchers and the jawbreakers? How is this situation different from the previous problem when we knew that there were 10 Jolly Ranchers and we wanted to know the number of Jawbreakers?) - Does everyone agree with way of thinking about the relationship? - What do we know so far about proportional relationships?	<i>What questions can I ask during the discussion that will help students keep the context and the goal of the problem in mind?</i> Possible Responses - Students should decide that there is 1 Jolly Rancher for every 2.6 Jawbreakers. <i>Ask a student to show this relationship with the counters.</i> - For every Jolly Rancher there are 2.6 Jawbreakers. - Jolly Ranchers increase at a consistent rate (1.0) and the Jawbreakers increase at a consistent rate (2.6). - I multiplied the number of Jolly Ranchers by 2 (5×2) then I did the same think to the number of Jawbreakers (13×2). In order to keep the same relationship what ever I do to one type of candy I have to do to the other type of candy. Asking students to repeat the reasoning or to put someone else's ideas into their own words will give everyone in the group an opportunity to hear the ideas several times and in different ways.
E X P L O R E	The next two problems will be presented simultaneously. They both follow the same Set-Up at the beginning of the lesson. Place the following tasks on the Overhead: a) Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers (5 to 13), but it contained 100 Jolly Rogers. How many Jawbreakers would you have? b) Suppose you had a candy jar with the same ratio of Jolly Ranchers to Jawbreakers (5: 13), but it contained 720 candies. How many of each kind of candy would you have?	Task a) is a “solve the proportion” task. One ratio and part of a second are given and the problem asks the value of the fourth number. Because of the scaffolding provided from the previous discussions in the lesson, students usually seem confident in their ability to solve this task. Task b) gives the total number of both candies and asks students to work backwards to find the number of both candies, keeping the 5:13 ratio. Working backwards tasks are often difficult for students.

PRIVATE PROBLEM SOLVING TIME

Give students 5 – 8 minutes of Private Time to begin to individually answer the questions.

FACILITATING SMALL GROUP PROBLEM SOLVING*What do I do if students have difficulty getting started?*

Now ask students to work in their groups to solve the problems. Assist students/groups who are struggling to get started by prompting with questions such as:

Task a)

-if you have 10 Jolly Ranchers, how many Jawbreakers will you have? How will this information help you solve the task?

Task b)

- What do you know, what do you need to know?
- How many total candies do you have with the initial ratio of 5:13?
- What does 720 have to do with the 5:13 ratio?
- What do you know so far about ratios and proportions?
- How can you use what you know about ratios and proportions to solve the task?
- What did you learn earlier that would help you with this task?

PRIVATE PROBLEM SOLVING TIME

Make sure that student's thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes.

FACILITATING SMALL GROUP PROBLEM SOLVING

What misconceptions might students have?

Misconceptions are common. Some strategies for helping students discover they have made an error include:

- Press students for the meaning of the numbers within the context of the problem, e.g., what does the 720 represent? What does the 100 represent?
- Described the problem in your own words. What does task a), b) ask you to figure out? How is this problem like the others that we discussed? What do you know about the relationship that might help you?
- What pictures do you have in your mind that will help you solve the problem?

What strategies might students use?

Task a)

- Students may continue work with the 1 : 2.6 ratio. Some may use the ratio to reason that “if 1 Jolly Rancher turns into 100 Jolly Ranchers that means it must have been multiplied by 100 so the 2.6 Jawbreakers also need to be multiplied by 100”.

- Students may build on the tables they started earlier:

5 Jrs 13 Jbs

10 Jrs 26 Jbs

15 Jrs 39 Jbs

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• •

- Students may think of the task in terms of using 20 as a multiple since most students realize 20 is a divisor of 100. In this case they would use the 5:13 ratio and multiply 5×20 and 13×20 .

What misconceptions might students have?

- Some students may still be struggling with the difference between proportional situations and additive ones. Students may not realize that although they may have added to find equivalent ratios, they did not add the same amount on both sides.
- Students may still not understand the need to keep the same rate when thinking proportionally.

What strategies might students use?

Using either the 1: 2.6 ratio or the 5:13 ratio and thinking about multiples that will increase the number of Jolly Ranchers to 100 indicates students have an understanding of how to find 100 Jolly Ranchers. If they then know to multiply the Jawbreakers by the same multiple, there is evidence they understand the multiplicative nature of proportional reasoning.

($\times 100$)

1 J.R.

100 J.R.

2.6 J.B.

260 J.B.

($\times 100$)

($\div 20$)

100 J.R.

5 J.R.

260 J.B.

13 J.B.

($\div 20$)

	Task b) - Students may build on the tables they started earlier. This is not the most productive way to solve the task, as the tables will be very long. Students usually add a third column to keep track of the total. - Students may figure that the 720 jar contains 40 times as many candies as the initial jar ($720 \div 18$) and see the need to multiply the 5 Jolly Ranchers and 13 jawbreakers by 40. - Students may begin a table but once they get to a number they can work with, such as 50:130 or 20: 52, they will use that information to find the solution.	If students bring up cross multiplication this a good place to look at the conceptual understanding attached to the procedure. $ \begin{array}{rcl} & (x40) & \\ 5 \text{ J. R.} & \longrightarrow & 200 \text{ J.R.} \\ \hline 18 \text{ total} & \longrightarrow & 720 \text{ Total} \\ & (x 40) & \end{array} $
S H A R E, D I S C U S A N D A N A L Y Z	<u>FACILITATING THE GROUP DISCUSSION</u> <i>What order will I have student share their solutions so I will be able to help them make connections between ratio and proportions?</i> Discuss Task a) and Task b) separately. As you circulate among groups of students, listen for how they are thinking about ratio and proportion. Mentally identify the students whose thinking you want the entire class to hear and in what order you want them to hear it. Ask students that have solutions based on multiplicative concepts, to share their thinking with the large group. Ask students that have used division to solve Task b) to share their thinking with the group.	<u>FACILITATING THE GROUP DISCUSSION</u> <i>What order will I have student share their solutions so I will be able to help them make connections between ratio and proportions?</i> As students share their solutions with the class, make sure the focus of the discussion is on the mathematics of ratio and proportion. Your reason for student sharing is to increase all students understanding of the meaning of ratio and proportion. During the group discussion, think about Accountable Talk moves such as: - Students put explanations given by their peers into their own words. This is a means of assessing understanding and provides other in the class with a secondary opportunity to learn. - Students are pressed to explain what they did and justify why their solution makes sense mathematically. - The teacher may re-voice student ideas if they will enhance understanding - Students build on other's ideas about ratio and proportion

E		Students look for similarities between the strategies presented and their mathematical thinking.
H O M E W O R K	Mom gave me 50 Jolly Ranchers and 125 Jawbreakers and asked me to make as many jars of candies as were possible with each jar containing 5 Jolly Ranchers and 13 Jawbreakers. How many jars could I make up? How many jawbreakers and how many Jolly Ranchers would be left over? Journal Prompt: Write down everything you know mathematically about ratio and proportion. Identify new mathematics learning.	This task asks students to think in terms of a multiplicative constant. If students multiply they will need to know when to stop, if they divide they will need to know that the smaller number represents the solution.