

CONSERVING WATER

6th Grade

Overview: In this lesson, students are asked to determine whether Oak Park or Fern Woods has cut their water consumption the most. Data is given for Oak Park, and students work in groups to design a method for collecting a sample of data from Fern Woods using either random, systematic, or convenience sampling in order to determine the water consumption level of the households. Teams collect a sample of data and then calculate the range, mean, median and mode of their sample. They then discuss within the group, and across groups, the strengths and weaknesses of the various sampling methods and different sample sizes. They draw conclusions about which community has cut their water consumption the most, which will require that they decide whether to use the mean or median water consumption figures, which will likely produce conflicting conclusions. A schedule is given to illustrate how the task can be engaged in over a period of three days.

Task: Developed as an extension of the summary on page 484 of the sixth grade text – R. Larson, L. Boswell, T. Kanold, & L. Stiff (2001), *Mathematics: Concepts and Skills, Course 1*, McDougall Littell.

Goals:

- Students will solve the problem using a variety of strategies and representations.
- Students will develop an understanding of how to use different sampling methods.
- Students will demonstrate an understanding of the meaning of mean, median and mode and how they reflect different aspects of “average”.
- Students will develop an understanding of the advantages and disadvantages of different sampling methods.
- Students will develop an understanding of the advantages and disadvantages of small vs. large samples.
- Students will be able to apply their findings to develop a recommendation for collecting water consumption data and will explain and justify their recommendation.

Content Standards:

- SDAP 1.1** Students compute and analyze range, mean, median, and mode for data sets.
- SDAP 1.2** Students understand how additional data added to data sets may affect these computations of measures of central tendency.
- SDAP 2.2** Students will identify different ways of selecting a sample and which method makes a sample more representative for a population.
- SDAP 2.4** Students identify data that represent sampling errors and explain why the sample might be biased.

Building on Prior Knowledge: Fifth Grade Standards

- SDAP 1.1** Know concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

The Task: Which Community Will Win the Water Conservation Challenge?

The Mayor of the town of Crystal Springs wants to encourage families to conserve water. In April, he found that households in the communities of Oak Park and Fern Woods used a lot of water – the typical household in each community used 6,300 gallons of water per month. On May 1st the mayor made a challenge – the community that uses less water over the summer will be able to host the fall carnival.

The newspaper stated that Oak Park cut their monthly household water consumption to 5,900 gallons of water during the month of May. Miguel, Sam, Mary and Tamara live in Fern Woods and their school asked them each to poll households in their community to find out how they are doing. Unfortunately, they don't have enough time or money to visit every home to gather their data. How could they collect information from a sample of homes in Fern Woods to find out whether they are on the way to winning the challenge, or whether they have to work harder to catch up with Oak Park?

Miguel decided to use systematic sampling, Sam decided to use convenience sampling, and Tamara decided to use random sampling to gather their data, but they need your help. With your team, select a task card to find out whom you will help. Work together to decide how you will use their method to collect your data to decide whether families in Fern Woods used less water than families in Oak Park during the month of May. Once you have selected the homes that will be in your sample, consult the "Home Visit" sheet to collect your data. Calculate the range, mean, median, and mode of your sample.

- Which of these results best represents the "typical" household water consumption in your sample? Do you think this result represents the "typical" water consumption in Fern Woods? Which community do you think uses less water, Fern Woods or Oak Park? Produce a poster that describes your methods and shows your results and conclusions. Explain your reasoning.

COMPARISON:

Mary collected her data using a self-selected sample. She placed a notice on Broad Street, but only five families responded. Her data was: 4,800, 5,300, 4,300, 6,400, and 4,800 gallons. She performed her calculations and put her findings in the summary chart. Add your team's results and conclusions to the summary chart.

- Compare your findings to those of the other teams. What do you think is the best way to determine *Which Community Conserved more Water?* Be prepared to explain your recommendations.

Extension Questions, if students need additional challenge:

- Would it be important to find out whether Oak Park reported their median or mean water consumption? Explain.
- Will it always be true that mean water consumption is higher than median water consumption? Explain.
- Why do you think some households use a lot more water than others?
- What can you do to make sure you are not wasting water at home?

Homework: Prepare a letter to send to the Mayor of Crystal Springs to recommend the best sampling procedure to use to collect water consumption data from Fern Woods. Be sure to mention the suggested sample size, sampling method, possible bias, and the measure you should use to determine the "typical" household water consumption. Justify your recommendations.

Project: Find out how much your utility company charges for water. Figure out how much money your family would save over a month, and over a year, if your family can use 1000 less gallons of water per month.

To solve this task successfully, students will need to understand how to apply a given method to select a sample from a given population. Students will need to know how to compute the range, mean, median and mode of their data. More importantly, they will need to understand that the different measures of central tendency have different characteristics, and that one may be more appropriate than the others for a given set of data. Students should notice that extreme scores in a sample influence the mean, but not the median. They should be able to argue for whether to use the mean or the median of their sample to represent the typical family. They should also see that some samples might be biased because of the sampling methods used.

NOTE:

The term “**typical**” was used instead of “**average**” in order to contextualize the idea of central tendency. Also, students often think that “average” always refers to “mean”, while “average” actually refers to any measure of central tendency including median and mode. The water consumption of 5,900 gallons for Oak Park was selected because it falls between the mean and median of the population data for Fern Woods, and thus students could argue for whether the mean or median would be the best measure to use. While the mode is the most *common* response, the data was selected to help students see that it may not be *typical*. Students may also realize that to be fair, they would need to find out which measure of central tendency was used by the other community (one of the extension questions).

Materials: Copies of Task Cards – make sure that several teams represent each of the three “students”; a summary data sheet for the entire class on chart paper or overhead; “Home Visit” sheet to be consulted once students have picked the homes in their sample. **For each team:** Water Conservation task; a Map of Fern Woods (without the water consumption data); one copy of “Selecting a Sample from a Population”; calculators, paper and markers for student presentations.

- Determine student teams prior to the lesson so that students who complement each other’s skills and knowledge core are working together.
- Arrange the desks so that students are in groups of 2 or 3. Place materials at each grouping.
- Smaller sized teams will allow for many different samples to be taken so that there will be a variety of results and conclusions; you will need at least six teams so that you will have multiple examples of each sampling method. Smaller groups will also give each child an opportunity to contribute.

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P	<u>HOW DO YOU SET UP THE LESSON?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.	<u>HOW DO YOU SET UP THE LESSON?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.
	<u>SETTING THE CONTEXT FOR THE TASK</u> Ask students to follow along as you read the task:	<u>SETTING THE CONTEXT FOR THE TASK</u> It is important that students have access to solving the problem from the beginning.
	The Mayor of the town of Crystal Springs wants to encourage families to conserve water. In April, he found that households in the communities of Oak Park and Fern Woods used a lot of water – the typical household in each community used 6,300 gallons of water per month. On May 1st the mayor made a challenge – the community that uses less water over the summer will be able to host the fall carnival. The newspaper stated that Oak Park cut their monthly household water consumption to 5,900 gallons of water during the month of May. Miguel, Sam, Mary and Tamara live in Fern Woods and their school asked them each to poll households in their community to find out how they are doing. Unfortunately, they don't have enough time or money to visit every home to gather their data. How could they collect information from a sample of homes in Fern Woods to find out whether they are on the way to winning the challenge, or whether they have to work harder to catch up with Oak Park? (continued)	• Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. • Also, have copies of the problem, the map of Fern Woods, and the description of the sampling methods available for each group. After having each team send a student to collect their task card, check students' understanding of the task by asking several students what they are asked to do, and what they are trying to find when solving the problem. Make sure that they have selected their task card and mention their sampling method when describing what they are to do. Tell students that they will have a few minutes to develop a strategy before working with their teammate(s).

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P S E T U P S E T U P	<u>SETTING THE CONTEXT FOR THE TASK (cont'd)</u> Miguel decided to use systematic sampling, Sam decided to use convenience sampling, and Tamara decided to use random sampling to gather their data, but they need your help. With your team, select a task card to find out whom you will help. Work together to decide how you will use their method to collect your data to decide whether families in Fern Woods use less water than families in Oak Park during the month of May. Once you have selected the homes that will be in your sample, consult the "Home Visit" sheet to collect your data. Calculate the range, mean, median, and mode of your sample. - Which of these results best represents the "typical" household water consumption in your sample? Do you think this result represents the "typical" water consumption in Fern Woods? Which community do you think uses less water, Fern Woods or Oak Park? Produce a poster that describes your methods and shows your results and conclusions. Explain your reasoning. <u>Comparison:</u> Mary collected her data using a self-selected sample. She placed a notice on Broad Street, but only five families responded. Her data was: 4,800, 5,300, 4,300, 6,400, and 4,800 gallons. She performed her calculations and put her findings in the summary chart. Add your team's results and conclusions to the summary chart. - Compare your findings to those of the other teams. What do you think is the best way to determine <i>Which Community Conserved more Water?</i> Be prepared to explain your recommendations.	Make sure the students develop their strategy and then select the homes in the sample from the street map first. Then they can consult the Home Visit sheet to collect the data. This models the way in which data is collected in real world settings. The sample should be selected without consideration of the results that they are likely to obtain. This task can be completed over several days. Day 1: Design the sampling procedure and select the sample. Begin making team poster, providing a written description of the sampling procedure. Day 2: Collect the data from the sample and calculate the range, mean, median and mode of the samples. Decide which result to use to represent the "typical" family. Add this to the team poster. Team presentations Day 3: Add team results to Class Summary Chart. Compare results across teams, compare sampling methods, the impact of sample size, discuss whether mean or median should be used; discuss extension questions, if possible.

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S E T U P S E T U P S E T U P	<u>SETTING THE CONTEXT FOR THE TASK (Cont'd.)</u> <i>COMPARISON:</i> As groups of students complete the task and can explain the thinking and reasoning behind their solutions, ask them to work on the "Comparison" part of the task.	<u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u> Setting up and reinforcing these expectations on a continual basis will result in them becoming a norm for the mathematics classroom. Eventually, students will incorporate these expectations into their habits of practice for the mathematics classroom. <u>INDEPENDENT PROBLEM-SOLVING TIME</u> It is important that students be given private think time to understand the problem for themselves and to begin to solve the problem in a way that makes sense to them.
	<u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u> Remind students that they will be expected to: • begin the task individually before consulting teammates. • explain their thinking and reasoning to others. • justify their solutions in the context of the problem. • make sense of other students' explanations. • ask questions of the teacher or other students when they do not understand. • use correct mathematical vocabulary, language, and symbols. Tell students that their team will be expected to share their solutions with the whole group using chart paper, the overhead projector, etc. They will also enter their findings into the summary table.	
	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> Circulate among the class as students work privately on the problem. Allow students time to individually make sense of the task before having students work as a team.	

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E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION</u> <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to find? • Why are we using a sample and not the whole population? • How will you find out which community uses less water using the data in your sample?	<u>FACILITATING SMALL-GROUP EXLORATION</u> <u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method.
E X P L O R E	<u>Possible Solution Paths</u> Strategies will be discussed as well as the questions that you might ask students and the misconceptions that you might encounter. Representations of these solutions are included at the end of this document.	<u>Possible Solution Paths</u> ** Indicates key mathematical ideas in terms of the goals of the lesson. Questions should be asked based on where the learners are in their understanding of the concept. It is important that student responses are given both in terms of the context of the problem and in correct mathematical language.
E X P L O R E	A discussion of possible strategies will be organized by the phase of the problem solution. Students must first develop a strategy to select their sample and then collect the data for that sample, then calculate the range, mean, median and mode for that sample. Once they have found their results, they will be able to explore ideas related to the important concepts.	As you circulate, make sure students develop a way to indicate the home addresses in their sample, and a place to record the data from these homes. ** It is important that students are not given the actual water consumption data prior to selecting the houses in their sample so they don't modify their samples to include, or avoid, particular data. This will help to model actual sampling procedures in the real world.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
EXPLORE	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Developing a strategy to select the sample, collect the data, calculate the Range, Mean, Median and Mode of the sample</i>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Developing a strategy to select the sample, collect the data, calculate the Range, Mean, Median and Mode of the sample</i> <u>Possible misconceptions or errors:</u> - Students using a convenience sampling method may not recognize that they must first decide their location in the city. What is convenient for someone who lives on Broad Street will not be convenient for someone who lives on Palm Street. You might ask, “Does it make a difference where you are in the city?” - Students using a random sampling method may fail to develop a method that is truly random. Instead, they may skip different numbers of homes each time. You might ask, “Does each house have a chance to be chosen?” - Students using a random sampling method may fail to realize that you can not select the same house more than once, even if it appears in the sample. - Students using a systematic sampling method may fail to decide, ahead of time, the method that they will use. You might ask, “What pattern will you use to select the houses?” - Once students have collected the data for their sample, students may not realize that values that occur more than once in their sample must be repeated in their data set. You might ask, “What if the other family used 100 more gallons. What would you do with their data?” - Before determining the median, students may fail to first organize the data in numerical order. You might ask, “What if your numbers were arranged in another order. Should the median change or be the same?” ** Developing a strategy must occur prior to actually deciding which homes will be in the sample.
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E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> Ask questions such as: • How did you select the homes in your sample? • How do you know that your method is a random/systematic/convenience sampling method? • How did you decide how many homes you would select? • How did you find the range, mean, median, and mode?	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> Possible Student Responses • Students should be able to clearly describe the method that they used to select the homes (see the notes at the end of the lesson plan for additional suggestions). - o Students using random samples should be able to say how they decided, ahead of time, the technique for randomizing their search (i.e., putting numbers in a hat); - o Students using systematic samples should be able to state their method (i.e., we picked a house and then skipped three houses); - o Students using convenience samples should begin by stating where they “live” and thus which homes are closest to them. • Students should be able to explain how their method corresponds to the description of sampling procedures in the handout “Goal 2” from their textbook. • Students should provide a rationale that suggests that they’ve grappled with the competing goals of getting enough data, and saving time and money. • Students should be able describe their method. Be sure that they have organized their data in numerical order prior to finding the median.

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E X P L O R E E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Interpreting their results and conjecturing about which community uses less water.</i> Ask questions such as: • What is the problem asking you to find? • What community do you think uses less water?	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Interpreting their results and conjecturing about which community uses less water.</i> <u>Possible misconceptions or errors:</u> • Students might not recognize that the mean is unduly affected by extreme scores. You might ask, “If you hadn’t selected that home with a very high water consumption, do you think your mean would have been different?” • Students may not recognize that they have a choice whether to use the mean or the median. You might ask, “Do you think you could have chosen the mean/median instead of the median/mean?” Possible Student Responses • Students should state that they are trying to find out which community uses less water. It is important to keep the focus on this goal to help them understand that these procedures and measures help us to answer questions about populations and are only a means to an end. • Students’ conclusions should be based upon how the results for their sample compare with the 5,900 gallon typical family water consumption in Oak Park. They may have to deal with conflicting measures – the mean and median could well lead to different conclusions.

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EXPLORE EXPLORE	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> • How did you decide this? • Why did you decide to use the <u>mean or median</u> (whichever they mention)? • Why didn't you use the mode (if there were duplicate water consumptions in their sample)?	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> • Students should be able to summarize the process they used. They should mention that they took a sample of the entire population; they should also indicate whether they used the mean or median value. • Students should be able to provide a rationale for their choice. They may say that: the mean uses all of the values and shows that some of the families use a lot more water than most; the median tells you about the "family in the middle" – half of the community uses more water, half uses less. • Students should state that in this task the mode is not a useful measure. Even though more than one family has the same water consumption, it still doesn't represent the typical water consumption of the entire sample. In this case they could see that the mode might be one of the lowest or highest values in the sample.

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S H A R E D I S C U S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> What solution paths will be shared and in what order and why? First have several teams present their team posters which should describe their sampling method, show their data, calculations, and conclusions. Make sure to include some teams that selected small samples and some that selected large samples. Make sure to include at least one example of each sampling method and to place the posters in a visible location so that students can refer to them later. Also select teams whose data sets produced divergent results. Teams will present their results and conclusions with the class, and will be questioned by the teacher and their classmates. If some teams were able to move to the COMPARISON question, a discussion of that question – which involves a comparison of team results – would follow. <u>Possible Solutions to be Shared</u> Because there will be a variety of combinations of small vs. large sample size and sampling method used, the following questions should be tailored to each specific case. <i>Team Presentations</i> Ask questions such as: • What were we trying to find out? What did your team do to answer the question?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> What solution paths will be shared and in what order and why? The order in which solutions are shared can assist the teacher in making certain the goals of the lesson are achieved. Starting with a solution path used by most or many of the students allows students access to the discussion. <u>Possible Solutions to be Shared</u> <i>Team Presentations</i> Possible Student Responses • Students should be able to restate the problem – they were trying to find out which community uses less water, Fern Woods or Oak Park. They needed to take a sample of the families in Fern Woods because they did not have enough time or money to go to every home. They should then describe their sampling method, including how they randomized or systematized their selection process and what measure they used to determine “typical” family water consumption.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> - Which community uses less water? How did you reach your conclusion? - Why did you use the mean/median to reach your conclusion? - Why did you sample _____ homes? <i>Questions for whole class – individual teams/students respond based upon their data</i> Ask questions such as: - What would happen to your median if the highest water consumption in your sample was 12,000 gallons?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> - Students should be able to use their team results to generalize about the community. They must state which sampling method they used and whether they used the mean or median. ** Again, keep the focus on the purpose of the problem – to reach a conclusion about which community uses less water. When different teams present different conclusions, an atmosphere for mathematical discourse is created. - It is important for students to share their rationale for using the mean or the median (see note below). ** Neither the mean nor the median would be an incorrect choice, but students could state that the mean is affected by extreme scores while the median indicates the middle value, regardless of what the extremes are. Either can be a desired characteristic, depending on the context. - Students should be able to justify their sample size, again referring to limiting the size of the sample because of limited time and money (therefore they could not get data from all of the homes) while there is also a need to get enough data to so that the sample represents the whole city (therefore visiting two or three homes would be clearly too few). <i>Questions for whole class – individual teams/students respond based upon their data</i> Possible Student Responses - Students should state that the median would not be changed if the highest water consumption was even higher because there would still be the same number of high scores and low

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S H A R E D I S C U S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> If you had sampled four more homes, would that have affected your mean, median, or mode? Explain <i>COMPARISON:</i> If a student or group of students complete the task and can explain the thinking and reasoning behind their solutions, ask them to work on the "Comparison" part of the task. <i>Comparing the Findings of All of the Teams (including Mary's data) contained in Summary Chart.</i>	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Students should recognize that the answer to this question would depend on the water consumptions of those four homes. They could say that the median wouldn't change if two of the water consumptions were high and two were low; the mean would not change if the high water consumptions and the low water consumptions balanced each other; the mean would be higher (lower) if some of the water consumptions were extremely high (low); the mode would have changed if the new water consumptions matched other water consumptions already in the sample. <i>Comparing the Findings of All of the Teams (including Mary's data) contained in Summary Chart.</i> <u>Important mathematical ideas:</u> ** Students may not understand that different samples will produce different results, and that none of them are "wrong". Looking at the class summary results will help them to see that a sample is only an approximation of the population. ** Students might fail to notice patterns in the class results – smaller samples tend to produce more variable results than larger samples, and convenience samples also tend to produce less reliable results than systematic or random samples. It is important to spend time exploring these patterns in the summary results. ** Students might notice that rearranging the data according to sample size or sampling method might make these relationships clearer. It might also be helpful to return to the specific data samples used by different teams to understand some of the "unusual" results (e.g., means that are much higher or lower than others).

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Ask questions such as: • Why do you think your team's results are different from the results of some of the other teams? • Compare the small samples to the large samples. Do you see any patterns? Why do you think this happened? • Do you think your sampling method produced the best results, or do you think another one would have been better? Why?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Possible Student Responses: • Students will immediately notice that there are differences in the results. They should state that different samples would produce different results. They might want to refer to some of the other team's raw data, and may notice that some of the samples contained several families that used a lot of water, or some of the samples contained families with very similar water consumptions. They might also notice that self-selected and convenience samples were more likely to produce unusual results than random or systematic samples. • Students should notice that the small samples tended to produce more varied results than large samples. • Students may tend to favor their own method. However, encourage them to focus on their results, as compared to those of the class. While theirs might have been easier (small sample, convenience sample), they must consider whether or not their method was likely to give them a good representation of the population data. ** While samples are taken in order to save time and money, very small samples often produce misleading results. Larger samples were more likely to contain the full range of values than smaller samples and thus will provide a better representation of the population.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Do you think any of the sampling methods produced <i>biased</i> results? What do you think made them biased?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Students might state that Mary's <u>self-selected sample</u> was biased because only people with low water consumptions responded, and the <u>convenience samples</u> were biased because they only sampled people who lived on the same street, and water consumptions were very different on the three streets. ** Self-selected samples are often biased, since people who respond may be very different from people who don't respond. For example, the cafeteria wants to find out students' choice of lunch options and the "vegetarian's club" makes sure that all of its members turn in their questionnaires – since few of the other students responded, the results give a false impression that students don't like meat. ** Convenience samples can also be misleading. For example, the class president wants to find out where the 6th grade wants to go for their class fieldtrip and decides to ask all of her friends. Since she and her friends love miniature golf, they decide that the mini golf park would be a great choice. Other students don't agree, but they weren't asked.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> • What sampling method do you think would be best to use to collect a sample of the water consumptions in Fern Woods? How large do you think the sample should be (assuming you can not gather all of the data)? Provide justification. <i>Extension Questions, if students need additional challenge:</i> • Would it be important to find out whether Oak Park reported their median or mean water consumption? • Will it always be true that mean water consumption is higher than median water consumption? • Why do you think some households use a lot more water than others? • What can you do to make sure you are not wasting water at home? <u>Homework:</u> Prepare a letter to send to the Mayor of Crystal Springs to recommend the best way to collect a sample of water consumption data from Fern Woods. Be sure to mention the suggested sample size, sampling method, possible bias, and the measure you should use to determine the “typical” household water consumption. Justify your recommendations.	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> • The class should debate various points of view, providing reasons. It is acceptable if the class does not reach a consensus. The homework assignment will allow students to share their individual views. <i>Extension Questions, if students need additional challenge:</i> • Students should state that this would be important to know. To be <i>fair</i>, both communities should use the same measure. • Mean water consumption will only be higher than median water consumption when the data has a few extreme cases on the high end; if the extreme cases are on the low end the mean will be lower than the median. • Some households can contain more family members than others; some may have a pool or use a lot of water on their lawns. • You can save water by taking short showers, not leaving the water running when you brush your teeth or rinse the dishes, etc. ** In order to make a <i>fair</i> comparison, both communities should use the same measure. For example, if one reports the mean and the other reports the median, it may be misleading to compare results.

A Note about Sampling Methods

Students should decide how large a sample they want to take prior to selecting the homes in their sample. The map does not contain water consumption data so that students will be selecting “addresses” of the homes in their sample. They can then consult the “Home Visit” sheet to collect their actual data. Encourage students to design a way to make a record of the homes they have selected, and the water consumption of those homes. They might want to share their system of recording on their team poster.

Random Sampling

You may need to first have a discussion about what “random” means. In a random sample, each item has the **same chance** of being selected as any other item. A common method for taking a random sample is to write each address on a slip of paper and put the papers into a hat. Students will then stir up the papers and pick out the number of addresses needed and will not return the selected papers to the hat. Students should be familiar with this method since it is often used for raffles.

Some students may find more efficient ways – they can have one box that has 3 slips of paper “Broad Street”, “Palm Street” and “Olive Street”. Another box would contain slips of paper with the numbers 10 through 19. In this case, students would first select a street and then an address. *It is important that they realize that for this method they will have to return the slips to the box* so that other homes on the same street or with the same house number would have a chance to be selected. By keeping track of the addresses selected (they might circle them on their map), they will know if they have selected the same home twice, in which case they will need to make another selection.

Systematic Sampling

A systematic sample is not random since once you have created a selection system (i.e., selecting all of the even number houses), then there are some homes that cannot be selected (e.g., 13 Broad Street). However, it is easier to collect a systematic sample than a random sample, and there is a greater chance to find representative data than with a convenience or self-selected sample.

You might notice, however, that the data set was designed so that students who decide to select even or odd numbered homes will actually be finding biased data. Families living in even-numbered homes in the population have higher water consumptions than odd-numbered homes. If students are perplexed by differences in the class results, you might encourage them to look at the population data.

TASK CARDS

MIGUEL

Systematic sample

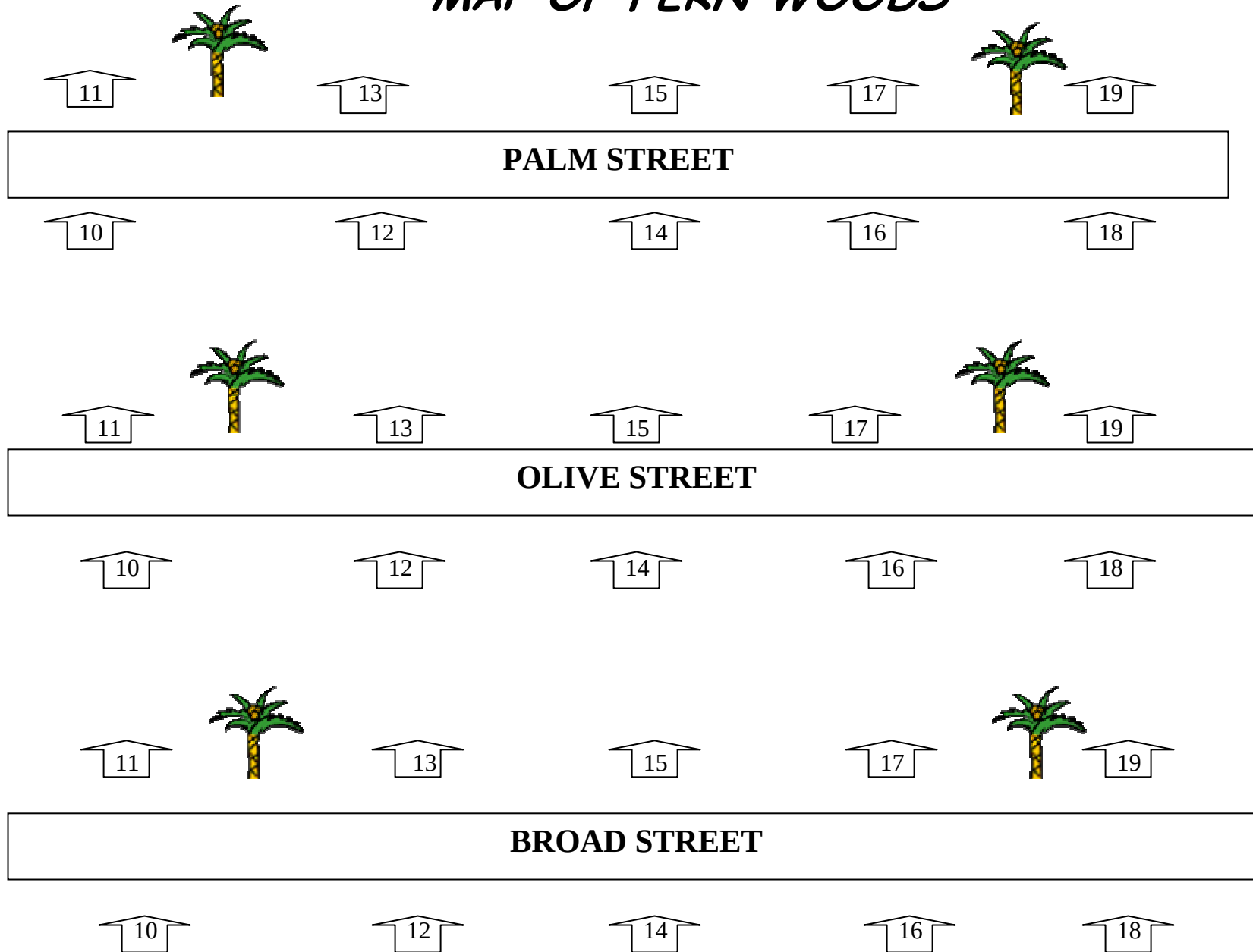
SAM

**Convenience
sample**

TAMARA

Random sample

MAP OF FERN WOODS



HOME VISIT SHEET FOR FERN WOODS*

WATER CONSUMPTION FOR MAY

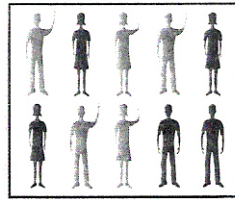
BROAD STREET			OLIVE STREET			PALM STREET	
House #	Water consumption (gallons per month)		House #	Water consumption (gallons per month)		House #	Water consumption (gallons per month)
10	4,300		10	5,300		10	5,900
11	4,900		11	8,300		11	5,800
12	4,800		12	4,700		12	6,200
13	6,900		13	9,300		13	13,000
14	4,300		14	5,300		14	5,600
15	8,300		15	6,800		15	10,400
16	5,300		16	5,100		16	6,300
17	6,400		17	7,600		17	10,400
18	5,100		18	5,100		18	5,700
19	7,100		19	5,200		19	11,400

*(to be consulted by teams **ONLY AFTER** they have decided which homes are in their sample)

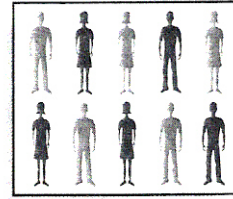
Goal 2 SELECTING A SAMPLE FROM A POPULATION

There are several ways to select a sample from a human population.

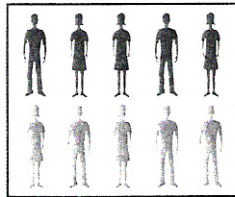
With *self-selected sampling*, you let people volunteer to be part of the sample.



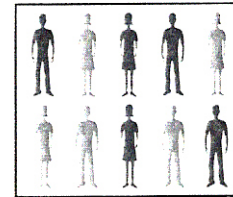
With *systematic sampling*, you use a pattern to select people, such as choosing every other person.



With *convenience sampling*, you choose easy-to-reach people, such as those in the first row.



With *random sampling*, each person has an equally likely chance of being chosen.



CLASS SUMMARY DATA

WHICH COMMUNITY USES LESS WATER?

[illegible]