

Fraction of a Fraction – 6th Grade

Overview: In this lesson, students use different strategies to model and reason finding a fraction ($\frac{1}{2}$ and $\frac{1}{3}$) of another fraction (in this case a mixed number $2\frac{1}{4}$).

Mathematics: To solve this task successfully, students must relate different representations of fractions and derive the meaning of taking a fraction of another fraction and its representation using squares, words, and numbers. Doing this allows students to make meaningful connections among the representations. It is more meaningful than just applying algorithms such as “multiply the numerators and denominators,” which mask the underlying structure and concepts.

Goals:

- Students will solve the problem using a variety of strategies.
- Students will interpret $\frac{1}{2}$ of $2\frac{1}{4}$, $\frac{1}{3}$ of $2\frac{1}{4}$, represent them as fractions of squares, and represent them with numbers.
- Students will compute a 20% discount.
- Students will describe how their representations show $\frac{1}{2}$ and $\frac{1}{3}$ of $2\frac{1}{4}$.
- Students will justify their solutions to the problem.

Number Standards:

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, earned interest, and tips.

Building on Prior Knowledge (from Q2):

Number Standards:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations.

Materials: Discounting Brownies task (attached), chart paper, graph paper, markers

Phase	Action	Comments
S E T U P	Prior to the lesson: • arrange the desks so that students are in groups of 4. • determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. • place materials for the task at each grouping. • solve the task yourself.	Students will be more successful in this task if they understand what is expected in terms of group work and the final product. It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different strategies for solving the problem will be given.
	<u>HOW DO I SET-UP THE LESSON?</u> Ask students to follow along as you read the problem. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem.	<u>HOW DO I SET-UP THE LESSON?</u> As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate the goal is to determine and demonstrate what $\frac{1}{2}$ of $2\frac{1}{4}$ and $\frac{1}{3}$ of $2\frac{1}{4}$ are. Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving.

E X P L O R E	<u>PRIVATE PROBLEM SOLVING TIME</u> Give students 5 - 7 minutes of private think time to begin to solve the problem individually. Circulate among the groups assessing students' understanding of the idea below. <u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> As you circulate among the groups, press students to show $\frac{1}{2}$ of $2\frac{1}{4}$, $\frac{1}{3}$ of $2\frac{1}{4}$ in more ways than one. After explaining their initial solutions you might say Now look at the squares again. Is there another way to represent those fractions? Could you use a different representation? <u>What do I do if students have difficulty getting started?</u> Allow students to work in their groups to solve the problem. Assist students/groups who are struggling to get started by prompting with questions such as: - Can you draw a picture of how much Mr. Vargas will get? - What would you need to show first? - Can you think of a simpler problem you could solve first? - Can you show a picture of taking $\frac{1}{2}$ of $\frac{1}{2}$? - How is this brownies problem different or similar to finding $\frac{1}{2}$ of $\frac{1}{2}$?	<u>PRIVATE PROBLEM SOLVING TIME</u> Make sure that students' thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes. <u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> The teacher's role when students are working in small groups is to circulate and listen with the goal of understanding students' ideas and asking questions that will advance student work. <u>What do I do if students have difficulty getting started?</u> By asking a question such as "Can you think of a simpler problem you could solve first?" the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine.
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What misconceptions might students have?

Look for and clarify any misconceptions students may have.

- a. Not understanding $2\frac{1}{4}$ as the whole. **How can you show how much of the brownies is left in the pans? Is that more or less than one pan? Will one half of what is left be more or less than one pan? Will one third of what is left be more or less than one pan? How do you know?**

Which problem-solving strategies might be used by students?

How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will approach the problem using a variety of strategies. Some strategies are shown below. Questions for assessing understanding and advancing student learning are listed for each.

Using a drawing

I. showing $2\frac{1}{4}$ and shading $\frac{1}{2}$ of that.

What does it mean to shade one half of your original picture? How do you know into how many pieces you need to divide a pan of brownies? Is one third of $2\frac{1}{4}$ going to be smaller or bigger than one half? How do you know? Are fractions related to percents? How?

What misconceptions might students have?

Misconceptions are common. Students may have learned one definition of fractions or one algorithmic way of multiplying fractions.

A strategy for helping students gain a better understanding is to:

- Ask students to use other representations of the same problem and find connections among them.

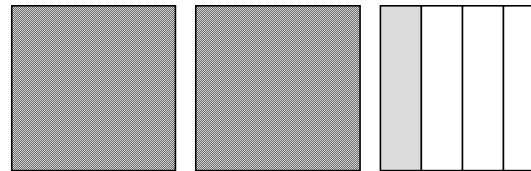
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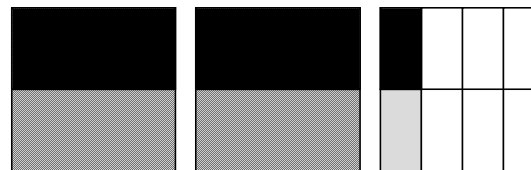
Using a drawing

I. showing $2\frac{1}{4}$ and shading $\frac{1}{2}$ of that.

Part A:



There are $2\frac{1}{4}$ pans of brownies left.



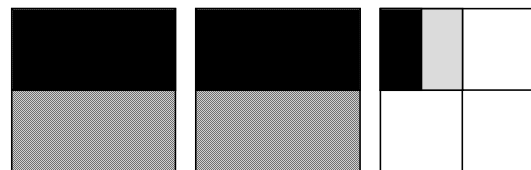
Mr. Vargas wants $\frac{1}{2}$ of the $2\frac{1}{4}$ of brownies left.

II. showing $2\frac{1}{4}$ and shading $\frac{1}{3}$ of that.

What does it mean to shade one half of your original picture? How do you know into how many pieces you need to divide a pan of brownies? When comparing two fractions how can you tell which one is bigger?

Or, $\frac{1}{2} + \frac{1}{2} + \frac{1}{8} = 1\frac{1}{8}$ of a pan.

Students may show a variation of the picture above:



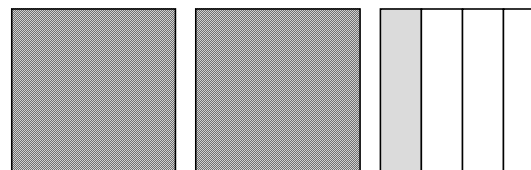
This would cost \$27.00 (1 pan = \$24.00 + $\frac{1}{8}$ of a pan = \$3.00)

before the discount. Paulo and Paula would have to charge Mr. Vargas 20% off \$27.00 (or 80% of \$27.00) that is \$21.60.

II. showing $2\frac{1}{4}$ and shading $\frac{1}{3}$ of that.

Part B:

There are $2\frac{1}{4}$ of brownies left.



Mr. Vargas wants $\frac{1}{3}$ of the $2\frac{1}{4}$ of brownies left.



Or, $\frac{1}{3} + \frac{1}{3} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$ of a pan.

Three fourths of a pan of brownies would cost \$18.00 ($\frac{3}{4}$ of

\$24.00). Mr. Vargas can afford it because with the 20% discount he would only have to pay \$14.40 (20% off \$18.00 or 80% of \$18.00).

B.. *–reasoning by multiplying*

Is multiplying by $\frac{1}{2}$ the same as dividing by two? How can you show that? Is multiplying by $\frac{1}{3}$ the same as dividing by 3? How can you show that? Could you represent what you have done with pictures?

B.. *–reasoning by multiplying*

Part A:

$$\frac{1}{2} \times 2\frac{1}{4} = \frac{1}{2} \times \frac{9}{4} = \frac{9}{8} = 1\frac{1}{8} \text{ pans of brownies.}$$

$$2\frac{1}{4} \div 2 = \frac{2}{2} + \frac{\frac{1}{4}}{2} = 1 + \frac{1}{8} = 1\frac{1}{8}$$

Part B:

$$\frac{1}{3} \times 2\frac{1}{4} = \frac{1}{3} \times \frac{9}{4} = \frac{9}{12} = \frac{3}{4} \text{ of a pan of brownies.}$$

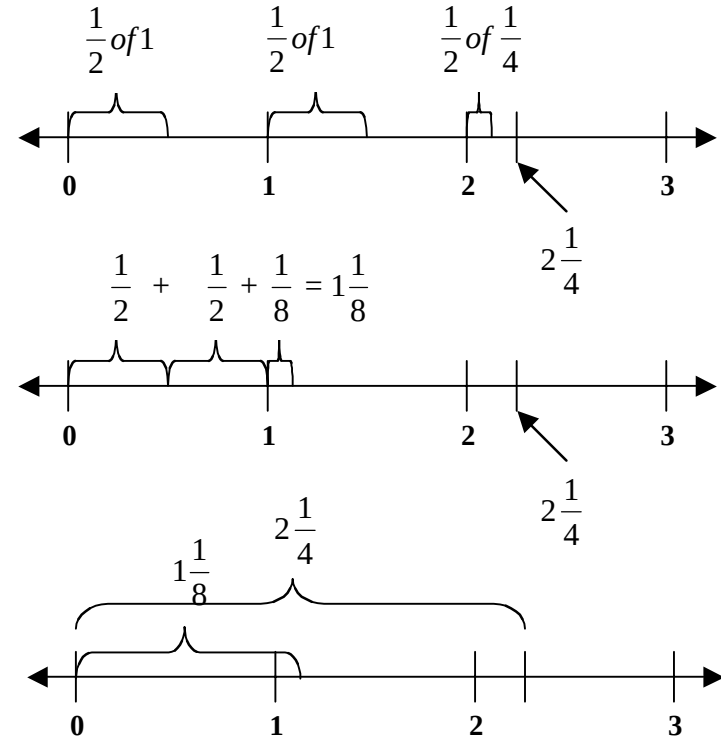
$$2\frac{1}{4} \div 3 = \frac{2}{3} + \frac{\frac{1}{4}}{3} = \frac{2}{3} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$$

Refer to pictures on page 5 to see pictorial representations

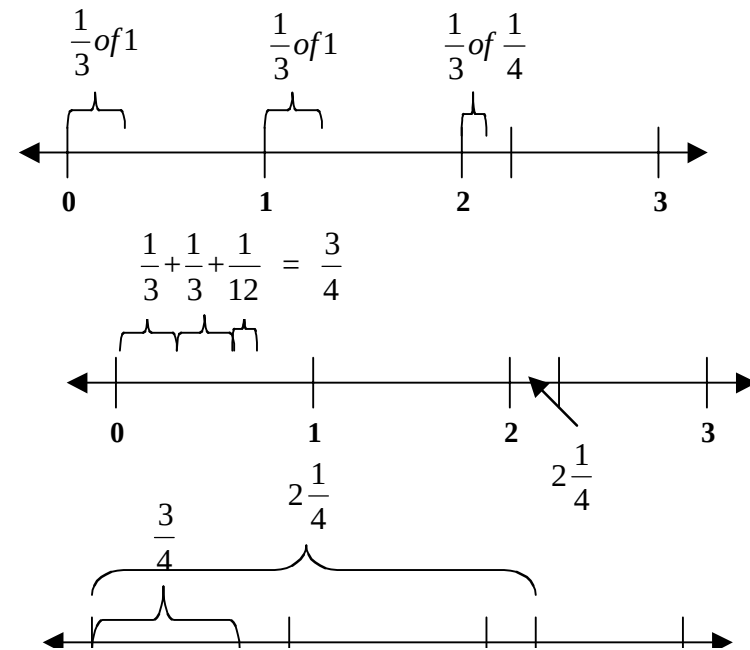
C. *– representing the fractions on a number line.*

C. – representing the fractions on a number line.

How did you know where to place $2\frac{1}{4}$? How do you know where to place any fraction? Why is adding one half of one, one half of one, and one half of one fourth the same as finding one half of $2\frac{1}{4}$?



How did you know where to place $2\frac{1}{4}$? How do you know where to place any fraction? How did you find what one third of one fourth was? Why is adding one third of one, one third of one, and one third of one fourth the same as finding one third of $2\frac{1}{4}$?



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FACILITATING THE GROUP DISCUSSION

What order will I have students post solution paths so I will be able to help students make connections between the solution paths?
As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of their representations.

Ask students to post their work in the front of the classroom.

The goal is to discuss mathematical ideas associated with finding a fraction of a mixed number.

What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? (Driving Questions

FACILITATING THE GROUP DISCUSSION

What order will I have students post solution paths so I will be able to help students make connections between the solution paths?
Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group. For this particular problem it could be best

What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? Driving questions ()*
Driving Questions have been provided because they will help to stimulate student interest, maintain the focus of the discussion on the problem context, and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or problem solving strategies.

D A N A L Y Z E	*) <i>Solutions using drawings</i> Ask students to present their solution and explain how their drawings help them to get the answers. You might ask Where in your drawing is $\frac{1}{2}$ of $2\frac{1}{4}$ represented? Where in your drawing is $\frac{1}{3}$ of $2\frac{1}{4}$ represented? How did you find out what portion of the pan was $\frac{1}{2}$ of $\frac{1}{4}$? How did you find out what portion of the pan was $\frac{1}{3}$ of $\frac{1}{4}$? Could solve these problems using multiplying by $\frac{1}{2}$ or by $\frac{1}{3}$? Is that represented on the drawings? How? How might you write a multiplication problem to find the amount of brownies Mr. Vargas would get? <i>Solution using multiplication:</i> How are these number sentences related to the drawings? Where on the drawings can you see the $\frac{9}{4}$ you multiplied by? You showed that the product of two fractions is the product of the numerators over the product of the denominators ($\frac{1}{2} \times \frac{9}{4} = \frac{9}{8}$ and $\frac{1}{3} \times \frac{9}{4} = \frac{9}{12}$). Why does that technique work? <i>Solution using number line</i> How are the representations on the number line related to the other two ways to solve this task? Where, on the number line, do we find the $\frac{9}{4}$ we multiplied by? What about the results of the multiplications ($\frac{9}{8}$ and $\frac{9}{12}$)?	<u>Accountable Talk</u> <i>Pointing to the drawing</i> Asking student to point to the drawing and showing where $\frac{1}{2}$ and $\frac{1}{3}$ of the pan of brownies are represented. <i>Repeating or Paraphrasing Ideas</i> Ask other students to put explanations given by their peers into their own words. This is a means of assessing understanding and providing others in the class with a second opportunity to hear the explanation. <i>Position-Driven Discussion</i> Press students to take a position and to support their claims with evidence from the drawings. Students must say why their drawings represent that fraction of the pan of brownies. In doing so they will have to provide reasons for their claims.
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Fraction of a Fraction: Discounting Brownies

Your task: Read the situation below and use pictures, diagrams, words, numbers, and/or symbols to determine and show how much brownies Mr. Vargas will buy and how much he will pay.

Paulo and Paula are tending the brownie booth at the school fair. The brownies are baked in square pans, and they are sold as fractional parts of a pan. A whole pan of brownies costs \$24.00. The cost any fractional part of a pan is that fraction of \$24.00. The school fair was almost over. Paulo and Paula wanted to sell all the remaining brownies in a hurry, so they decided to offer a discount of 20% on all sales. They had $2\frac{1}{4}$ pans of brownies left. Remember they originally sold a pan of brownies for \$24.00.

Part A. Mr. Vargas offered to buy half of all that they had left.

1. How much will Mr. Vargas purchase?
2. How much should Paulo and Paula charge Mr. Vargas?

Part B. When Mr. Vargas got his bill, he realized he had only \$20.00 in his wallet, so he said, “I guess I’ll only buy $\frac{1}{3}$ of what you have left.”

1. Now how much will Mr. Vargas buy?
2. Can he afford this much? Explain your reasoning.