

Gauging Gas Mileage

7th Grade

Overview: In this task, students determine which car is more fuel efficient. Students must first use a written description and map to determine how far each car traveled and then investigate data about the number of gallons of gas used by each car to determine which is more fuel efficient.

Mathematics: To solve the task successfully, students may approach it by determining the rate – miles per gallon, gallon for “x” miles, or gallons per mile. Although miles per gallon is the most common rate used when discussing fuel efficiency, other rates, such as gallon for “x” miles or gallons per mile, will enrich the discussion and push students to a deeper understanding of rates.

Goals:

- Determine, apply, and interpret the concept of rates.
- Understand rates as a multiplicative comparison of two measures.
- Explain and justify one’s thinking and reasoning.
- Apply and interpret different strategies for solving the same problem.

Seventh Grade Content Standards:

MG 1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

AF 4.2 Solve multi-step problems involving rate, average speed, distance, and time or direct variation.

Building on Prior Knowledge: Sixth Grade Content Standards

NS 1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour).

Materials: Gauging Gas Mileage (attached), calculators

Adapted from “Comparing Fuel Economy”, pp. 38-39, *Comparing and Scaling*, **Connected Mathematics**, Prentice Hall, 2002.

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P	<u>HOW DO YOU SET UP THE LESSON?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.	<u>HOW DO YOU SET UP THE LESSON?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.
S E T U P	SETTING THE CONTEXT FOR THE TASK Ask students to follow along as you read the problem: After graduating from UCLA, Luis and Keira both got teaching jobs in Los Angeles. They each bought a new car for commuting to work, and one afternoon they had a friendly argument about whose car was better. Luis claimed his car was more fuel-efficient. Keira challenged him to prove his claim. Since they would both be traveling home for Thanksgiving, Luis suggested they use the trip to test their gas mileage.	SETTING THE CONTEXT FOR THE TASK It is important that students have access to solving the problem from the beginning. • Have the map displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. • Make certain that students understand the vocabulary used in the problem (i.e. "fuel-efficient" means using a less amount of fuel, or gasoline, over the same distance or going a longer distance on the same amount of gas). This term, as well as others that may cause confusion to students, could be posted on a word wall.
S E T U P	Luis and Keira are from different cities in northern California, Merced and San Francisco. But they both traveled the first part of their trips on Interstate 5 to get to their homes. Luis then traveled on to Merced while Keira traveled to San Francisco. After Thanksgiving, they compared how fuel efficient their cars were. Luis made the trip to Merced and back using 27.8 gallons of gasoline. Keira used 32.2 gallons of gasoline on her trip to San Francisco and back. Luis claimed his car was more fuel efficient but Keira disagreed. 1. Whose car was more fuel-efficient? Explain how you know and why you think your answer is correct. 2. Would it make sense to use percents to settle the argument between Luis and Keira? Explain your reasoning.	• Check on students' understanding of the task by asking several students what they know and what they are trying to find when solving the problem. • Be careful not to tell students how to solve the task, or to set up a procedure for solving the task, because your goal is for students to do the problem solving.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P S E T U P	SETTING UP THE EXPECTATIONS FOR DOING THE TASK Remind students that they will be expected to: • justify their solutions in the context of the problem. • explain their thinking and reasoning to others. • make sense of other students' explanations. • ask questions of the teacher or other students when they do not understand. • use correct mathematical vocabulary, language, and symbols. Tell students that their groups will be expected to share their solutions with the whole group using the board, the overhead projector, etc.	SETTING UP THE EXPECTATIONS FOR DOING THE TASK Setting up and reinforcing these expectations on a continual basis will result in them becoming a norm for the mathematics classroom. Eventually, students will incorporate these expectations into their habits of practice for the mathematics classroom.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> Circulate among the groups as students work privately on the problem. Allow students time to individually make sense of the problem.	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> It is important that students be given private think time to understand the problem for themselves and to begin to solve the problem in a way that makes sense to them.
	<u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to find? • What information do you know about Luis? About Keira? • Look at the map. Where did Luis travel? Where did Keira travel? • If you travel 150 miles on 10 gallons of gasoline, and your friend travels 200 miles on 10 gallons of gasoline, whose car is more fuel-efficient?	<u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer, or that do not explicitly suggest a solution method. **Students' previous experience with problems of this type may have resulted in them using a scale factor approach. If students try to use this method, ask them why this problem does not lend itself to such an approach (i. e., the numbers in the problem would make it very difficult to find a scale factor.)
	<u>Possible misconceptions or errors:</u> • Students might claim that Luis's car is more fuel-efficient since he went a shorter distance or because he used less gasoline. To help them understand that being fuel-efficient depends on the amount of fuel used for a particular distance, you might give an example. • Students who have heard of the term "miles per gallon" may confuse the number of gallons of gasoline used with the miles per gallon. As a result, they may believe Keira's car is more fuel-efficient since she used 32.2 gallons. You might ask, "Why do you say Keira's car is more fuel efficient? What does the '32.2 gallons' mean?"	<u>Possible misconceptions or errors:</u> It is important to have students explain their thinking before assuming they are making an error or have a misconception. After listening to their thinking, ask questions that will move them toward understanding their misconception or error.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> - Students might use the mileage for the trip to each city rather than using the round-trip mileage. Ask them to reread the part of the problem that states “to Merced and back..” and “to San Francisco and back...” <u>Possible Solution Paths</u> Strategies will be discussed as well as the questions that you might ask students. Representations of these solutions are included at the end of this document. <i>Finding the rate – miles per gallon:</i> If students are focusing on the number of miles per gallon each car gets, ask questions such as: If one friend’s car travels 150 miles on 10 gallons of gasoline and another friend’s car travels 200 miles on 10 gallons of gasoline, whose car is more fuel efficient? Why? How far would each car travel on one gallon of gasoline? How could you find out whether Luis or Keira’s car is more fuel efficient? Can you find the solution a different way? Make certain that students use correct labels: Explain how you found 23.8 (or 22.0)? What does the 23.8 (or 22.0) represent?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <u>Possible Solution Paths</u> Questions should be asked based on where the learners are in their understanding of the concept. It is important that student responses are given both in terms of the context of the problem and in correct mathematical language. <i>Finding the rate – miles per gallon:</i> Possible Student Responses - Students should state that the second friend’s car is more fuel efficient since it can travel farther on the same 10 gallons of gas. - Student should say that the first car would travel 15 miles on one gallon of gas while the second would travel 20 miles. - Students should state their method for arriving at the answer and should explain that they were trying to find how many miles each car could travel on one gallon of gas. Students should use the term “miles per gallon”.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Finding the rate – gallon for every “x” miles:</i> NOTE: This solution is very similar to finding miles per gallon. Students are focusing on one gallon of gas and how many miles each car can travel. If students are focusing on one gallon of gas, ask questions such as: • If one friend’s car uses 15 gallons of gas to travel 150 miles and a second friend’s car uses 10 gallons to travel the same distance, whose car is more fuel-efficient? Why? • One gallon of gas, would allow the cars to travel how many miles? • How could you find out whether Luis or Keira’s car is more fuel-efficient? • Can you find the solution a different way? Make certain that students use correct labels: • Explain how you found 23.8 (or 22.0)? What does the 23.8 (or 22.0) represent?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Finding the rate – gallon for every “x” miles:</i> Possible Student Responses • Students should state that the second friend’s car is more fuel-efficient since she uses less gasoline to travel 150 miles. • Student should say that the first car would use one gallon of gas to go 10 miles, while the second car would use one gallon of gas to go 15 miles. • Students should state their method for arriving at the answer and should explain that they were trying to find, for one gallon of gas, how many miles each car could travel. Students should use the term “one gallon for every 22 miles” or “one gallon for every 23.8 miles.”

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Finding the rate – gallons per mile:</i> NOTE: This solution is different than finding gallons for every “x” mile. Students are focusing on how much gas would be used to travel one mile. Ask questions such as: • If one friend’s car uses 15 gallons of gas to travel 150 miles, and a second friend’s car uses 10 gallons of gas to travel 150 miles, whose car is more fuel-efficient? Why? • How much gas would each car use to travel one mile? • How could you find out whether Luis or Keira’s car is more fuel-efficient? • Can you find the solution a different way? Make certain that students use correct labels: • Explain how you found .045 or .042? What does the .045 or .042 represent? CONSIDER: If a student or group of students completes the task and can explain the thinking and reasoning behind their solutions, ask them to work on the “Consider” part of the task.	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Finding the rate – gallons per mile:</i> Possible Student Responses • Students should state that the second friend’s car is more fuel-efficient since she uses less gasoline to travel 150 miles. • Student should say that the first car would use .1 gallons of gas to go one mile while the second car would use .067 gallons of gas to go one mile. • Students should state their method for arriving at the answer and should explain that they were trying to find how much gas it would take to travel one mile. Students should use the term “.045 gallons per mile” or “.042 gallons per mile”.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> <u>Possible Solutions to be Shared</u> <i>Solution with a misconception or error:</i> You might begin by saying: In one of my other classes, someone said that Luis's car was more fuel-efficient because he used less gasoline. Is this correct? Why or why not? Can you give me an example?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> The purpose of the discussion is to assist the teacher in making certain the goals of the lesson are achieved by students. Questions and discussions should focus on the important mathematics and processes that were identified for the lesson. **Indicates key mathematical ideas in terms of the goals of the lesson. <u>Possible Solutions to be Shared</u> <i>Solution with a misconception or error:</i> Sometimes it is effective to start with a misconception held by many students so that it can be addressed and clarified. Possible Student Responses Students should state that using less gasoline does not necessarily mean a car is more fuel-efficient. If one car uses 10 gallons of gas to travel 100 miles while another uses 12 gallons to travel 240 miles, the second car is more fuel-efficient. **The importance of considering <u>both</u> the amount of gasoline used <u>and</u> the number of miles traveled is essential when discussing fuel efficiency, and is fundamental to the concept of rate in this problem.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Finding a rate – miles per gallon:</i> Ask questions such as: • Why did you divide the number of miles by the number of gallons? • What does your answer mean in the context of the problem? • So how far could each travel on 10 gallons of gas? On 20 gallons of gas? • Is there another way to solve the problem?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Finding a rate – miles per gallon:</i> Possible Student Responses • Students should be able to explain that it is difficult to compare whose car is more fuel-efficient just by looking at the distances traveled and number of gallons of gasoline used. They each traveled different distances and used different amounts of gas. • Students should explain that 22.0 miles per gallon means Luis can travel 22.0 miles on ONE gallon of gas while Keira can travel 23.8 miles on ONE gallon of gas. That is what the label, “miles per gallon” or “mpg”, means. • Students should state that Luis could travel 220 miles on 10 gallons of gas, while Keira could travel 238 miles – a difference of 18 miles. Luis would travel 440 miles and Keira 476 miles on 20 gallons of gas – a difference of 36 miles. **If you double the number of gallons of gas, the distance traveled also doubles. This is a key concept underlying rates – a multiplicative relationship.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Finding a rate – gallon for every “x” miles:</i> Ask questions such as: • Why did you divide the number of miles by the number of gallons? • What does your answer mean in the context of the problem? • So how far could each travel on 10 gallons of gas? On 20 gallons of gas? • How is this solution different from the “miles per gallon” solution? • Is there another way to solve the problem?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Finding a rate – gallon for every “x” miles:</i> Possible Student Responses • Students should be able to explain that it is difficult to compare whose car is more fuel-efficient just by looking at the distances traveled and number of gallons of gasoline used. They each traveled different distances and used different amounts of gas. • Students should explain that 1 gallon for every 22 miles means Luis can travel 22.0 miles on ONE gallon of gas while 1 gallon for every 23.8 miles means Keira can travel 23.8 miles on ONE gallon of gas. That is what the label, “miles per gallon” or “mpg”, means. • Students should state that Luis could travel 220 miles on 10 gallons of gas while Keira could travel 238 miles – a difference of 18 miles. Luis would travel 440 miles and Keira 476 miles on 20 gallons of gas – a difference of 36 miles. **If you double the number of gallons of gas, the distance traveled also doubles. This is a key concept underlying rates – a multiplicative relationship. • Students should explain that “miles per gallon” means the number of miles traveled for every gallon of gas while “gallon per miles” means one gallon of gas allows you to travel a certain number of miles. **Both solutions make use of rates by comparing the number of gallons of gas and the distance traveled.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Finding a rate – gallons per mile:</i> Ask questions such as: • Why did you divide the number of gallons by the distance? • What does your answer mean in the context of the problem? • So how much gas would each of them use to travel 100 miles? 1,000 miles? • How is this solution different from the “gallons for every “x” mile”?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Finding a rate – gallons per mile:</i> Possible Student Responses • Students should be able to explain that it is difficult to compare whose car is more fuel-efficient just by looking at the distances traveled and number of gallons of gasoline used. They each traveled different distances and used different amounts of gas. • Students should explain that Luis would use .045 gallons of gas to travel 1 mile, while Keira would use .042 gallons to travel 1 mile. • Students should state that Luis would use 4.5 gallons of gas to travel 100 miles, while Keira would use 4.2 gallons – a difference of .3 gallons. If traveling 1,000 miles, Luis would use 45 gallons of gas and Keira would use 42 gallons – a difference of 3 gallons. **As you increase the number of miles, the amount of gas used increases multiplicatively. (1,000 is ten times as much as 100 and 3 is ten times as much as .3). The multiplicative relationship is a key concept underlying rates. • Students should explain that “miles per gallon” means the number of miles traveled for ONE gallon of gas while “gallons per mile” means the amount of gas used to travel ONE mile. **Both solutions make use of rates by comparing the number of gallons of gas and the distance traveled.

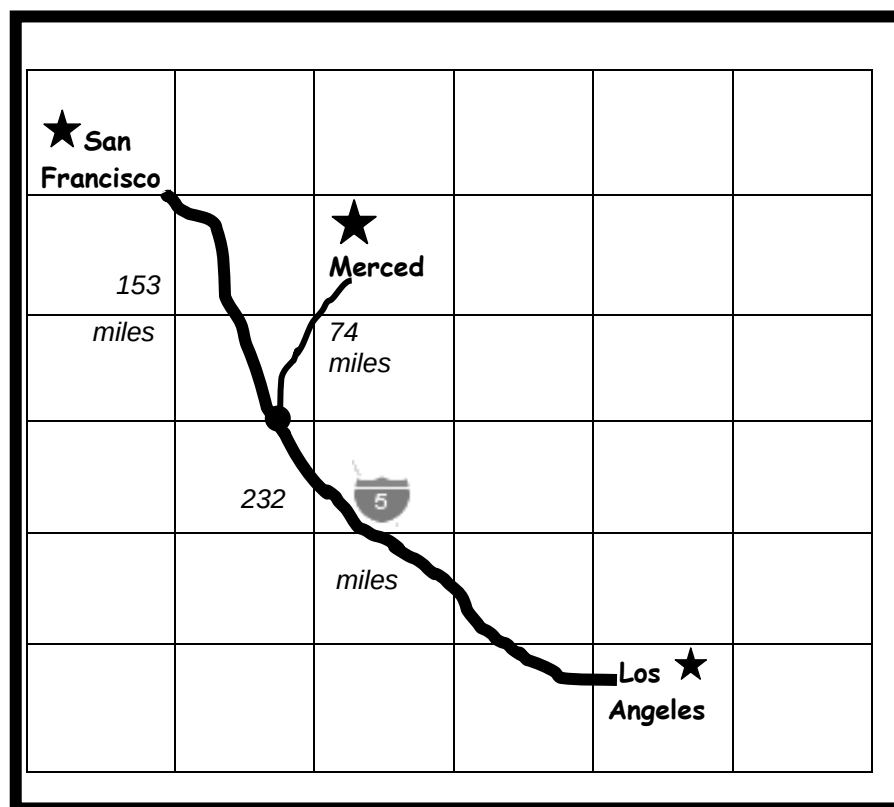
Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>CONSIDER:</i> Ask students to look at the “Consider” question: Many people travel 10,000 miles or more in their cars in one year. Describe how being fuel efficient would impact Luis and Keira if they traveled that amount in one year. <i>Assignment:</i> Two stores are having a sale on CD's. Music Mania is selling 5 CD's for \$37.99 and World of Music is selling 7 CD's for \$51.99. Which store's sale is a better value? Explain how you know.	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Possible Student Responses <i>See “Possible Solutions” for responses.</i> **Because of the multiplicative nature of rates, as the number of miles traveled increases, so does the <u>difference</u> between the amount of gas used. This is evident in the graph.

After graduating from UCLA, Luis and Keira both got teaching jobs in Los Angeles. They each bought a new car for commuting to work and one afternoon they had a friendly argument about whose car was better. Luis claimed his car was more fuel-efficient. Keira challenged him to prove his claim. Since they would both be traveling home for Thanksgiving, Luis suggested they use the trip to test their gas mileage.

Luis and Keira are from different cities in northern California, Merced and San Francisco. But they both traveled the first part of their trips on Interstate 5 to get to their homes. Luis then travels on to Merced while Keira travels to San Francisco. After Thanksgiving, they compared their fuel economy. Luis made the trip to Merced and back using 27.8 gallons of gasoline. Keira used 32.2 gallons of gasoline on her trip to San Francisco and back. Luis claimed his car was more fuel-efficient but Keira disagreed.

1. Whose car was more fuel-efficient? Explain how you know and why you think your answer is correct.
2. Would it make sense to use percents to settle the argument between Luis and Keira? Explain your reasoning.

Consider: Many people travel 10,000 miles or more in their cars in one year. Describe how being fuel-efficient would impact Luis and Keira if they both traveled that many miles in one year.



POSSIBLE SOLUTIONS:

Finding the rate – miles per gallon:

1. Luis

$$232 + 74 = 306 \text{ miles to Merced}$$

$$306 \times 2 = 612 \text{ miles to Merced and back}$$

Luis used 27.8 gallons of gasoline.

$$612 \text{ miles} / 27.8 \text{ gallons} = 22.0 \text{ miles per gallon}$$

Keira

$$232 + 153 = 385 \text{ miles to San Francisco}$$

$$385 \times 2 = 770 \text{ miles to San Francisco and back}$$

Keira used 32.2 gallons of gasoline.

$$770 \text{ miles} / 32.2 \text{ gallons} = 23.8 \text{ miles per gallon}$$

Since Keira's car got more miles per gallon, her car would be more fuel-efficient. She can go 23.8 miles on every gallon of gas, while Luis can only go 22.0 miles on one gallon of gas. So for every gallon of gas, she can go 1.8 more miles than Luis can.

2. Although percents could be used for this problem, it is not as efficient as finding a rate such as miles per gallon. (ex. Luis uses 27.8 gallons of gas to drive 612 miles, so he would use 4.5% of a gallon to drive one mile. Likewise, Keira uses 32.2 gallons of gas to drive 770 miles, so she would use 4.2% of a gallon to drive 1 mile.)

Finding the rate – gallon for “x” miles:

1. Luis

Keira

$232 + 74 = 306$ miles to Merced

$232 + 153 = 385$ miles to San Francisco

$306 \times 2 = 612$ miles to Merced and back

$385 \times 2 = 770$ miles to San Francisco and back

Luis used 27.8 gallons of gasoline.

Keira used 32.2 gallons of gasoline.

$612/27.8 = 22$. For every gallon of gasoline,
Luis can travel 22 miles.

$770/32.2 = 23.8$. For every gallon of gasoline,
Keira can travel 23.8 miles.

Since Keira's car can go further on one gallon of gasoline, her car would be more fuel-efficient. For every gallon of gasoline, Keira can travel 1.8 more miles than Luis.

2. Although percents could be used for this problem, it is not as efficient as finding a rate such as “one gallon for every x miles”. (ex. Luis uses 27.8 gallons of gas to drive 612 miles, so he would use 4.5% of a gallon to drive one mile. Likewise, Kiera uses 32.2 gallons of gas to drive 770 miles, so she would use 4.2% of a gallon to drive 1 mile.)

Finding the rate – gallons per mile:

1. Luis

$$232 + 74 = 306 \text{ miles to Merced}$$

$$306 \times 2 = 612 \text{ miles to Merced and back}$$

Luis used 27.8 gallons of gasoline.

$$27.8 \text{ gallons} / 612 \text{ miles} = .045 \text{ gallons per mile}$$

Keira

$$232 + 153 = 385 \text{ miles to San Francisco}$$

$$385 \times 2 = 770 \text{ miles to San Francisco and back}$$

Keira used 32.2 gallons of gasoline.

$$32.2 \text{ gallons} / 770 \text{ miles} = .042 \text{ gallons per mile}$$

Since Keira's car got less gallons per mile, her car would be more fuel-efficient. For every mile she travels, her car uses .042 gallon while for every mile Luis travels, his car uses .045 gallons. Keira's car uses .003 less gallons of gasoline for every mile traveled.

2. Although percents could be used for this problem, it is not as efficient as finding a rate such as gallons per mile. (ex. Luis uses 27.8 gallons of gas to drive 612 miles, so he would use 4.5% of a gallon to drive one mile. Likewise, Kiera uses 32.2 gallons of gas to drive 770 miles, so she would use 4.2% of a gallon to drive 1 mile.)

CONSIDER:

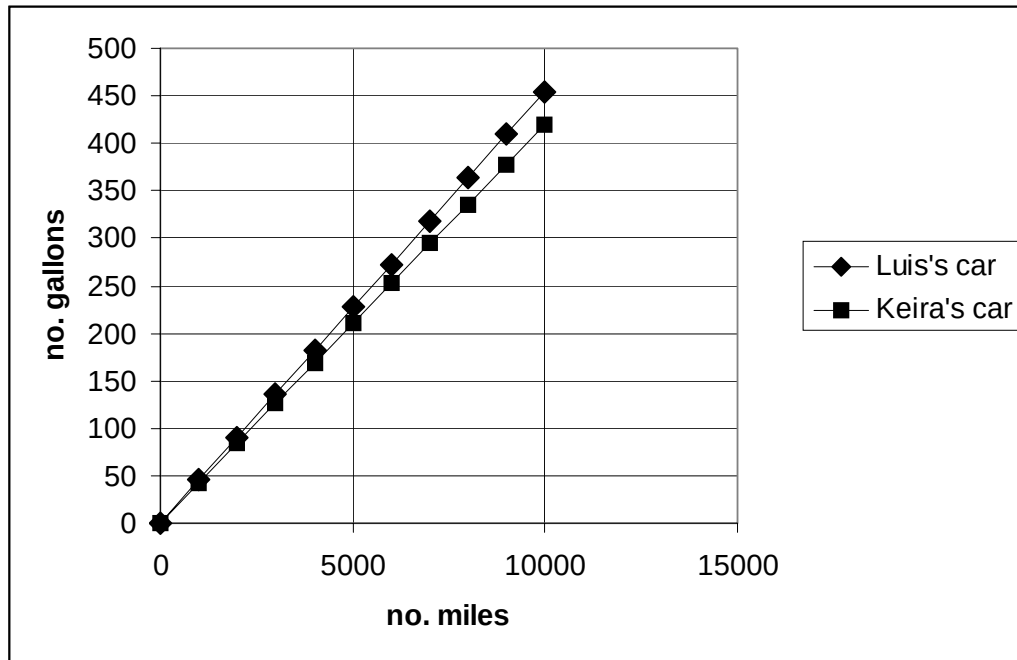
Luis's car got 22.0 miles per gallon while Keira's car got 23.8 miles per gallon. If they each traveled 10,000 miles in one year:

EXPRESSION:

Luis: $10,000/22 = 454.5$ gallons of gas in one year

Keira: $10,000/23.8 = 420$ gallons of gas in one year

Luis would use 454.5 gallons of gas in one year while Keira would use 420 gallons. Keira would use over 30 less gallons of gas in a year.

GRAPH:

In one year, Luis would use 34.5 more gallons of gas than Keira. At the current rate of \$3 for one gallon of gas, Luis would pay over \$100 more per year for gas than Keira.

*Note: If the terms "discrete" and "continuous" have been used to describe data previously, you could have a discussion as to why the data in this problem are continuous.