

Magic Rectangle 6th Grade Lesson Plan

LESSON OVERVIEW

Overview:

In this lesson, students are challenged to find all of the missing angles in a diagram involving a rectangle by measuring only one of the angles. In addition, they must explain and justify why it does not matter which angle in the diagram is measured, nor what the size of the rectangle is.

Mathematics in the Lesson:

To solve this task successfully, students must be able to measure angles and use their knowledge of complementary and supplementary angles and the sum of the angles of a triangle to determine missing angles in a diagram. In addition, they must be able to explain and justify why they only need to measure one angle in the diagram to determine the remaining angles and that it does not matter which angle they choose to measure.

Goals of the Lesson:

- To deepen students' understanding of complementary and supplementary angles and the sum of the angles of a triangle by applying this knowledge to solve for a variety of unknown angles.
- To develop/strengthen students' ability to use mathematical reasoning when solving problems.
- To develop/strengthen students' ability to explain and justify their thinking and support their argument with appropriate mathematical evidence.

Standards

MG 2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

MR 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

MR 1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

MR 2.5 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

Academic Mathematical Language

The following terms should be reinforced/developed throughout the lesson:

- supplementary angles
- segment
- right angle
- complementary angles
- perpendicular

Assumption of prior knowledge/experience:

- knowledge of supplementary and complementary angles
- knowledge that the sum of the angles of a triangle is 180 degrees
- knowledge of acute and obtuse angles

Materials

- task and task sheet
- protractors

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

6th Grade Unit 4 task
2005 – 2006

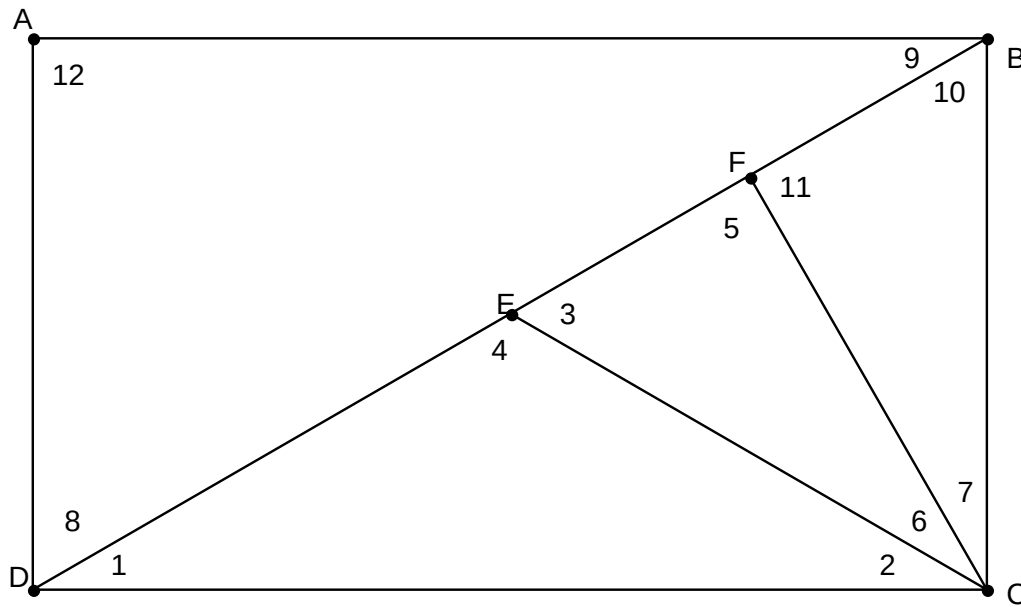
The "Magic" Rectangle

Mr. Wizard claims that he can perform “magic” with the rectangle shown below. He says that by measuring just one angle in the diagram that is not a right angle, he can use the magic to figure out all of the remaining angles. Your task is to:

1. find out if it is possible to figure out the measures of all of the angles in the diagram by just measuring one angle that is not a right angle.
2. explain to Mr. Wizard why his “magic trick” works. Be certain to use correct mathematical properties and vocabulary in your explanation.

Explore:

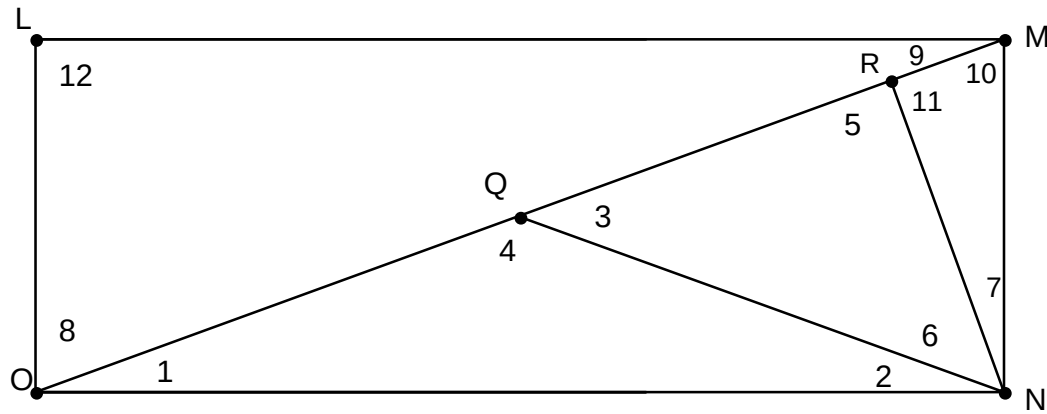
3. determine if this trick will work for all other “magic rectangles” that Mr. Wizard could draw (remember, a “magic rectangle” has to meet the special conditions).



**Special conditions for
Mr. Wizard's "Magic
Rectangles"**

- ABCD is a rectangle
- E is the midpoint of segment AC
- $\angle 1$ and $\angle 2$ have the same measure
- segment BF is perpendicular to segment AC

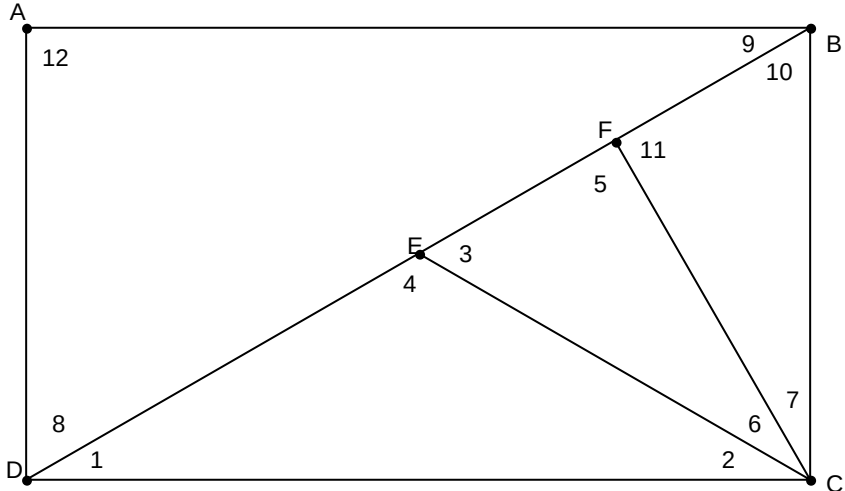
Another “Magic Rectangle)



Special conditions for Mr. Wizard’s “Magic Rectangles”

- LMNO is a rectangle
- Q is the midpoint of the line segment MO.
- $\angle 1$ and $\angle 2$ have the same measure
- line segment NR is perpendicular to line segment OM.

POSSIBLE SOLUTIONS:

	• Students should recognize that $\angle A$ is a 90 degree angle since ABCD is a rectangle. • Students should recognize that $\angle 11$ is 90 degrees since CF is perpendicular to BD. • Students should recognize that $\angle 5$ is a 90 degree angle because it is supplementary to $\angle 11$.
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- Students may then begin by measuring any of the other marked angles:

An example of beginning with 1:

- The measure of 1 is 30° so the measure of 2 is also 30° since we know they have equal measures.
- The measure of 8 is 60° since it is complementary to 1. We know that D is a right angle because ABCD is a rectangle.
- The measure of 4 is 120° because the sum of the measures of the angles of a triangle is 180° and the sum of the measures of 1 and 2 is 60° .
- The measure of 3 is 0° because it is supplementary to 4.
- The measure of 6 is 30° because the sum of the measures of the angles in a triangle is 180° and the sum of the measures of 3 and 6 is 90° .
- The measure of 7 is 30° because C is a right angle since ABCD is a rectangle and the sum of the measures of 2 and 6 is 60° .
- The measure of 10 is 60° because the sum of the measures of the angles in a triangle is 180° and the sum of the measures of the 90 degree angle and 7 is 120° .
- The measure of 9 is 30° because it is complementary to 10. We know that B is a right angle since ABCD is a rectangle.

When measuring other angles, similar explanations to those above should follow.

THE LESSON

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P	<u>HOW DO YOU SET UP THE TASK?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.	<u>HOW DO YOU SET UP THE TASK?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.
S E T U P	<u>SETTING THE CONTEXT FOR THE TASK</u> Ask students to follow along as you, or a student, read the problem. Mr. Wizard claims that he can perform magic with the rectangle shown below. He says that by measuring just one angle in the diagram that is not a right angle, he can use magic to figure out all of the remaining angles. Your goal is to: determine if it is possible to find the measures of all of the angles in the diagram by just measuring one angle that is not a right angle. determine if this "magic trick" will work for other rectangles. describe to Mr. Wizard what you have discovered about his "magic trick" and explain to him why it works. Be certain to use correct mathematical properties and vocabulary in your explanation.	<u>SETTING THE CONTEXT FOR THE TASK</u> It is important that students have access to solving the problem from the beginning. - Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. - Check on students' understanding of the task by asking several students what they know and what they are trying to find when solving the problem. - Be careful not to tell students how to solve the task because your goal is for students to do the problem solving.
S E T U P	Tell students that they may measure their angle to the nearest 5 degrees.	

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E X P L O R E	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> It is important that students be given private think time to understand and make sense of the problem for themselves and to begin to solve the problem in a way that makes sense to them.	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> • Tell students to work on the problem by themselves for a few minutes. • Circulate around the class as students work individually. Clarify any confusions they may have, but do not tell them how to solve the problem.
E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to find? • Do you know the measures of any of the angles in the diagram? • What kinds of geometric figures do you see in the diagram? What do you know about them?	<u>FACILITATING SMALL-GROUP EXPLORATION</u> Tell students they may now work with their partners. As students continue working, circulate around the classroom. <u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method.
E X P L O R E		

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E X P L O R E E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <u>Possible misconceptions or errors:</u> - Incorrectly using the protractor to measure the angle and getting an angle measure that is the supplement of the actual angle. Does the angle measurement you found make sense? Is the angle acute or obtuse? How does this compare to the measure that you found? - Failing to recognize that a rectangle has 4 right angles. Tell me what you know about rectangles. Can we know the measurement of any of the angles without using a protractor?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <u>Possible misconceptions or errors:</u> It is important to have students explain their thinking before assuming they are making an error or have a misconception. After listening to their explanation, ask questions that will move them toward understanding their misconception or error.

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E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u>
	<u>Possible Solution Paths</u> Strategies will be discussed as well as the questions that you might ask students. Students can begin by measuring any angle, other than the right angles. ** Indicates important questions in terms of getting at the mathematical goals of the lesson. Make statements and ask questions such as: • Identify all of the angles we need to find. What are some angles we already know and that we would not have to measure? • **Measure one of the angles. What do you know about angles and triangles that will help you to solve this problem?	<u>Possible Solution Paths</u> • Questions should be asked based on where the learners are in their understanding of the concept. • It is important that student responses are given both in terms of the context of the problem and in correct mathematical language. ** Indicates key mathematical ideas in terms of the goals of the lesson Possible student responses: • Students should recognize that ABCD is a rectangle and has 4 right angles and that the measure of $\angle BFC$ is 90 degrees since CF is perpendicular to BD. Students should also recognize that the measure of $\angle 5$ is 90 degrees since it is supplementary to BFC. **Press students to explain <u>why</u> the 2 angles are supplementary and what that means. • **Students should recognize the following depending on which angle they measured. It is important that they can explain <u>why</u> they know: * $\angle 1$ and $\angle 2$ have the same measure. Once one of them is measured, they can determine the measure of $\angle 4$ by using the sum of the angles of a triangle. * $\angle 3$ is supplementary to $\angle 4$ because they make a straight angle and the measure of a straight angle is 180 degrees. * $\angle 1$ is complementary to $\angle 8$ and $\angle 9$ is complementary to $\angle 10$ because the angles of a rectangle are right angles. And therefore the sum of angles 1 and 8 must be 90 degrees and the sum of angles 9 and 10 must be 90 degrees. * The sum of the measures of angles 3, 6, and the 90 degree angle is 180 degrees because they make up $\triangle EFC$ and the sum of the measures of the angles of a triangle is 180 degrees.
	E X P L O R E	
E X P L O R E		

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E E X P L O R E		<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> * The sum of the measures of angles 8 , 9, and the 90 degree angle is 180 degrees because they make up $\triangle ABD$ and the sum of the angles of a triangle is 180 degrees. * The sum of the measures of angles 7 , 10, and the 90 degree angle is 180 degrees because they make up $\triangle BCF$ and the sum of the measures of the angles of a triangle is 180 degrees. * The sum of the measures of angles 1 , 10, and the 90 degree angle is 180 degrees because they make up $\triangle BCD$ and the sum of the measures of the angles of a triangle is 180 degrees.

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S H A R E D I S C U S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> What solution paths will be shared, in what order, and why? ** Indicates responses that get at the key mathematical ideas in terms of the goals of the lesson. Possible Solutions to be Shared You might begin by asking what students found to be the measure of angle DEC (angle 4 on page 4). Then list what different students had. The majority should have found approximately the same measure. Once there is some agreement on the measure of this angle, ask students which angle they measured. Write down the different starting points. **How is it that you can start with different angles and still get the same answer? You could then continue the discussion by having one group or one student share their solution. Have them use a diagram on the overhead so that all students can follow along. Then have a group who began with a different angle share their solution. **Press students to use correct terminology and justify their reasoning with prompts such as:	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> The purpose of the discussion is to assist the teacher in making certain that the goals of the lesson are achieved by students. Questions and discussions should focus on the important mathematics and processes that were identified for the lesson. ** Indicates responses that get at the key mathematical ideas in terms of the goals of the lesson. Possible Solutions to be Shared ** Pressing students to explain and justify their reasoning will move them towards the mathematical reasoning goals of this lesson. **Students will have a variety of responses to this question. Listen for responses that address complementary angles, supplementary angles, and the sum of the angles of a triangle.

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S H A R E D I S C U S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> • How do you know the sum is 180 (or 90) degrees? • What does “supplementary angles” mean? • What does “complementary angles” mean? • How do you know that the angle measure is 90 degrees? You might also prompt other students to explain what was said: • Can someone else put in their own words what ____ was saying? • Did someone do this problem a different way? After having students explain their thinking, ask the question: • I wonder if this magic trick will work for other rectangles? Ask students what they think and to give a reason why. • Then pose the following problem: Let's look at another rectangle and see if this magic trick works. (Refer to the second rectangle.) This time start with an angle different from the angle you measured in the last problem, and find all of the remaining angles in the diagram.	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> • Students should be able to explain how they determined the measures of angles using the correct terminology. They should be able to state what supplementary and complementary angles are. They should also state that the sum of the measures of the angles of a triangle is 180 degrees. • Press students to justify why they think the trick either will or will not work with other rectangles. • Students should arrive at the same conclusion as they did for the previous problem.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Have one group or one student who began by measuring an angle different from those measured in the previous problem share their solution. Have them use a diagram on the overhead so that all students can follow along. Then have a group who began with a different angle share their solution. **Press students to use correct terminology and justify their reasoning. Conclude the lesson by having students answer the last question: Explain and justify to Mr. Wizard what you have discovered about his “magic trick”. Why does his “trick” always work? Be certain to use correct mathematical properties and vocabulary in your explanation.	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> - In their explanations, student should include the following: * The measures of angles 1 and 2 always equal each other. Therefore, they will always be able to find the measure of angle 4 by using the sum of the angles of a triangle. * Angles 8 and 1 and angles 9 and 10 will always be complementary because they form a right angle. * Angle 5 will always measure 90 degrees because it is supplementary to the 90 degree angle. * The sum of the measures of the angles in a triangle is 180 degrees.

TASK SHEETS

