

## Problems with Percents

### 7<sup>th</sup> Grade

**Overview:** In this lesson, students determine who is correct about the price of a pair of jeans that are on sale after already being reduced in price. The problem is a contextual problem that uses several diagrams to display important information.

**Task:** Problems with Percents (adapted from “Making Sense of Percents”, *MTMS*, Sept. 2003)

**Goals:**

- Students will solve the problem using a variety of strategies and representations.
- Students will develop an understanding of how taking multiple discounts on an item changes its price.
- Students will explain and justify their solutions to the problem.

**Content Standards:**

**NS 1.7** Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

**Building on Prior Knowledge:**

**NS 1.2** Add subtract multiply and divide rational numbers

**NS 1.3** Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

To solve this task successfully, students need to understand how to calculate the percent of a given number. They also need to understand that when an item is discounted, the amount of the discount is subtracted from the original price and results in a sale price.

**Materials:** Problems with Percents Task (attached); calculator

- Determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together.
- Arrange the desks so that students are in groups of 3 or 4. Place materials at each grouping.

**Note:** Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

| Phase                     | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
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| S<br>E<br>T<br><br>U<br>P | <p><b><u>HOW DO YOU SET UP THE LESSON?</u></b></p> <p>Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | <p><b><u>HOW DO YOU SET UP THE LESSON?</u></b></p> <p>It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|                           | <p><b><u>SETTING THE CONTEXT FOR THE TASK</u></b></p> <p>Ask students to follow along as you read the problem.</p> <p><b>Julie and her mother are shopping for some new jeans for school. They notice a rack of jeans with this sign on top of it: "40% discount on ticketed price of already reduced merchandise." Julie finds a pair of jeans on the rack, but unfortunately part of the price tag has been torn off. The tag looks like this: "\$<del>50</del> reduced 25% to..." Julie's mom claims that they can take 65% off the original price to determine the cost of the jeans. Julie claims that her mother is incorrect. Who is right – Julie or her mom? How do you know? What price will they pay for the jeans?</b></p> | <p><b><u>SETTING THE CONTEXT FOR THE TASK</u></b></p> <p>It is important that students have access to solving the problem from the beginning.</p> <ul style="list-style-type: none"> <li>• Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem.</li> <li>• Discuss what the term, "discount," means. Many students will know this but the term may not be familiar to all students. Make certain they understand that a discount is subtracted from a price to get a sale price.</li> <li>• If there are other words that may be confusing to students (e.g., "reduced", "ticketed", etc.), take time to discuss what those words mean in the context of this problem.</li> </ul> <p>Check on students' understanding of the task by asking several students what they are trying to find when solving the problem.</p> |

| Phase                     | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                           |
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| S<br>E<br>T<br><br>U<br>P | <p><b><u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u></b></p> <p>Remind students that they will be expected to:</p> <ul style="list-style-type: none"> <li>• justify their solutions in the context of the problem.</li> <li>• explain their thinking and reasoning to others.</li> <li>• make sense of other students' explanations.</li> <li>• ask questions of the teacher or other students when they do not understand.</li> <li>• use correct mathematical vocabulary, language, and symbols.</li> </ul> <p>Tell students that their groups will be expected to share their solutions with the whole group using chart paper, the overhead projector, etc.</p> | <p><b><u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u></b></p> <p>Setting up and reinforcing these expectations on a continual basis will result in them becoming a norm for the mathematics classroom. Eventually, students will incorporate these expectations into their habits of practice for the mathematics classroom.</p> |

| Phase                                                                                                                                                       | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
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| <p><b>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</b></p> <p><b>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</b></p> <p><b>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</b></p> | <p><b><u>INDEPENDENT PROBLEM-SOLVING TIME</u></b><br/>Circulate among the groups as students work privately on the problem. Allow students time to individually make sense of the problem.</p> <p><b><u>FACILITATING SMALL-GROUP EXPLORATION</u></b><br/><u>What do I do if students have difficulty getting started?</u></p> <p>Ask questions such as:</p> <ul style="list-style-type: none"> <li>• <b>What are you trying to find?</b></li> <li>• <b>What does it mean to “take 40% off” of a price?</b></li> <li>• <b>How much did the jeans cost originally?</b></li> </ul> | <p><b><u>INDEPENDENT PROBLEM-SOLVING TIME</u></b><br/>It is important that students be given private think time to understand the problem for themselves and to begin to solve the problem in a way that makes sense to them.</p> <p><b><u>FACILITATING SMALL-GROUP EXPLORATION</u></b><br/><u>What do I do if students have difficulty getting started?</u></p> <p>It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method.</p> <p><u>Possible misconceptions or errors:</u><br/>Julie’s mom has a misconception about percents that is common with students, as well as adults. Taking a discount on a discounted price is not the same as simply adding the two discount rates and taking a discount on the original price. Ways to address this misconception will be given in the next section.</p> <p><u>Other misconceptions or errors might include:</u></p> <ul style="list-style-type: none"> <li>• not subtracting the discount from the price to get the sale price. Remind students what the word “discount” means.</li> <li>• incorrectly converting the percents to decimals or fractions (e.g., 40% = 4 or .04), particularly when entering them into a calculator. You might ask students, “<b>What does 40% mean? If you think of a square representing 100% and then shaded 40% of it, how much would be shaded? How does that compare to what you wrote (or put in you calculator)?</b>”</li> </ul> |

| Phase                                                                                          | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| <div data-bbox="176 261 296 521">EXPLORE</div> <div data-bbox="176 521 296 1016">EXPLORE</div> | <p><b><u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u></b></p> <p><u>Possible Solution Paths</u><br/>Strategies will be discussed as well as the questions that you might ask students and the misconceptions that you might encounter. Representations of these solutions are included at the end of this document.</p> <p><i>Adding the two percents to find the discount:</i></p> <p>Ask questions such as:</p> <ul style="list-style-type: none"> <li>• <b>Why did you add the percents?</b></li> <li>• <b>Let's try an easier problem. What if the first discount was 50% and the second discount was 50%? What would your discount rate be? Does this make sense? What would a discount of 100% mean?</b></li> <li>• <b>What if the first discount was 50% and the second discount was 75%? What would that mean if you used your method?</b></li> </ul> | <p><b><u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u></b></p> <p><u>Possible Solution Paths</u><br/>Questions should be asked based on where the learners are in their understanding of the concept.</p> <p>It is important that student responses are given both in terms of the context of the problem and in correct mathematical language.</p> <p><i>Adding the two percents to find the discount:</i></p> <p><b>Possible Student Responses</b></p> <ul style="list-style-type: none"> <li>• Students will probably incorrectly state that since there was a 25% discount and a 40% discount that made a total discount of 65%.</li> <li>• Students will probably say that it would be a 100% discount, which does not make sense. A 100% discount would mean the discount was the price of the item, which would make the item free.</li> </ul> <p>Students should state that using their method, the discount would be 125%, which is more than the price of the item.</p> |

| Phase                                                                                             | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
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| <div>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</div> <div>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</div> | <p><b><u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u></b></p> <p><i>Determining the first discount and sale price and using that result to determine the second discount and sale price:</i></p> <p>Ask questions such as:</p> <ul style="list-style-type: none"> <li>• <b>What do you think we would need to do first? How would you find the price of the jeans that should have been on the price tag?</b></li> <li>• <b>What does “reduced 25% mean”? How would you find the discount?</b></li> <li>• <b>Why do we need to find the 25% discount before we find the 40% discount? Why does it matter?</b></li> <li>• <b>What would happen to the price of the jeans if we found the 40% discount first and then found the 25% discount? How do you know?</b></li> <li>• <b>Is there another way to solve this problem?</b></li> </ul> | <p><b><u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u></b></p> <p><i>Determining the first discount and sale price and using that result to determine the second discount and sale price:</i></p> <p><b>Possible Student Responses</b></p> <ul style="list-style-type: none"> <li>• Students should say that they would find the 25% discount and then subtract it from the original price of the jeans.</li> <li>• Students might connect this to the concept of discount. If not, ask them what the word “reduce” means in general and what it would mean to reduce the price by a certain amount.</li> <li>• Students should realize that the 40% discount should be taken off the reduced price. Since the reduced price is less than the original price, the discount on the reduced price will be less than discounting the original price by 40%.</li> <li>• Students may initially believe that there would be a difference in price if the percents were switched. Ask them to show you what would happen if the percents were switched.</li> </ul> |

| Phase                                                                                             | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| <div>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</div> <div>E<br/>X<br/>P<br/>L<br/>O<br/>R<br/>E</div> | <p><b><u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u></b></p> <p><i>Determining the sale price without finding the discount:</i></p> <p>Ask questions such as:</p> <ul style="list-style-type: none"> <li>• <b>Why are you using those percents? What do they mean?</b></li> <li>• <b>Which part of the problem would you do first? Why?</b></li> <li>• <b>What would happen to the price of the jeans if we found the 40% discount first and then found the 25% discount? How do you know?</b></li> <li>• <b>Is there another way to solve this problem?</b></li> </ul> | <p><b><u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u></b></p> <p><i>Determining the sale price without finding the discount:</i></p> <p>Students who use this method realize that the sale price of an item is a percent of the original price that is <math>1 - \text{discount rate}</math>. (i.e. A discount rate of 25% means the sale price would be 75% of the original price.)</p> <p><b>Possible Student Responses</b></p> <ul style="list-style-type: none"> <li>• Students should say that the percents they are using are the percents the new price is of the previous price. A discount of 25% means the new price would be 75% of the previous price. A discount of 40% means the new price would be 60% of the previous price.</li> <li>• Students should say that they would first find the price missing from the tag because they need to take 60% of that price to find the final price.</li> </ul> <p>Students may initially believe that there would be a difference in price if the percents were switched. Ask them to show you what would happen if the percents were switched.</p> |

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| <b>S</b><br><b>H</b><br><b>A</b><br><b>R</b><br><b>E</b><br><br><b>D</b><br><b>I</b><br><b>S</b><br><b>C</b><br><b>U</b><br><b>S</b><br><br><b>A</b><br><b>N</b><br><b>D</b><br><br><b>A</b><br><b>N</b><br><b>A</b><br><b>L</b><br><b>Y</b><br><b>Z</b><br><b>E</b> | <p><b><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u></b><br/>           What solution paths will be shared and in what order and why?</p> <p><b><u>Possible Solutions to be Shared</u></b></p> <p><i>Adding the percents to find the discount:</i></p> <p>It might be interesting to begin with a group who began the problem incorrectly by adding the two percents. If you are concerned that this might embarrass a student, you might have this solution prepared ahead of time and say, <b>“When I did this problem with another class, one of the groups produced this solution.”</b></p> <p>Ask questions or make statements such as:</p> <ul style="list-style-type: none"> <li>• <b>How did you get 65% percent?</b></li> <li>• <b>Explain how you got your answer.</b></li> <li>• <b>Did anyone get a different answer? Why are the answers different?</b></li> <li>• <b>Let’s try an easier problem. What if the first discount was 50% and the second discount was 50%? If we use your method, what would your discount rate be? Does this make sense? What would a discount of 100% mean?</b></li> <li>• <b>What if the first discount was 50% and the second discount was 75%? What would that mean if you used your method?</b></li> </ul> | <p><b><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u></b><br/>           What solution paths will be shared and in what order and why?</p> <p>The order in which solutions are shared can assist the teacher in making certain the goals of the lesson are achieved.</p> <p><b><u>Possible Solutions to be Shared</u></b></p> <p><i>Adding the percents to find the discount:</i></p> <p>Starting with a solution that contains errors or misconceptions allows the misconceptions or errors to be addressed publicly in case other students or groups had the same misconception or error.</p> <p><b>Possible Student Responses</b></p> <ul style="list-style-type: none"> <li>• Students should say that they added the 40% and 25% to get the 65%.</li> <li>• Students will probably state they took 65% of \$50 to get a discount of \$32.50, and subtracted that from \$50 to get a sale price of \$17.50.</li> <li>• Students will begin to disagree about the answers. Move them to the next example.</li> <li>• Students should say that it would be a 100% discount, which does not make sense. A 100% discount would mean the discount was the price of the item, which would make the item free.</li> <li>• Students should state that using their method the discount would be 125%, which is more than the price of the item.</li> </ul> |

| Phase                                                                                                                  | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
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| Phase                                                                                                                                                                                                                                                                | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
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| <b>S</b><br><b>H</b><br><b>A</b><br><b>R</b><br><b>E</b><br><br><b>D</b><br><b>I</b><br><b>S</b><br><b>C</b><br><b>U</b><br><b>S</b><br><br><b>A</b><br><b>N</b><br><b>D</b><br><br><b>A</b><br><b>N</b><br><b>A</b><br><b>L</b><br><b>Y</b><br><b>Z</b><br><b>E</b> | <p><b><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u></b></p> <p><i>Determining the sale price without finding the discount:</i></p> <p>Ask questions such as:</p> <ul style="list-style-type: none"> <li>• <b>Why did you use those percents? How do they relate to the percents used by the other group?</b></li> <li>• <b>Using your method, how would you find the discounts?</b></li> <li>• <b>What would happen to the price of the jeans if we found the 40% discount first and then found the 25% discount? How do you know?</b></li> <li>• <b>Why does this work?</b></li> </ul> <p>If time permits, you might demonstrate, or have a student demonstrate, why the order in which the discount is taken does not matter by using variables and a formula.</p> <p>The final 'Consider' question might also be discussed or given as a homework assignment.</p> | <p><b><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u></b></p> <p><i>Determining the sale price without finding the discount::</i></p> <p><b>Possible Student Responses</b></p> <ul style="list-style-type: none"> <li>• Students should say that the percents they are using are the percents the new price is of the previous price. A discount of 25% means the new price would be 75% of the previous price. A discount of 40% means the new price would be 60% of the previous price.</li> <li>• Students should state that the first discount would be \$50 - \$37.50 or \$12.50. Similarly, the second discount would be \$37.50 - \$22.50 or \$15.00.</li> <li>• Ask students to show you what would happen if the percents were switched.<br/>(The point of this last question is for students to realize that because of the commutative property for multiplication, the order in which the discounts are taken does not matter.)</li> </ul> |

| Phase                                                                                                                  | TEACHER PEDAGODY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY                                                    |
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| S<br>H<br>A<br>R<br>E<br><br>D<br>I<br>S<br>C<br>U<br>S<br>S<br><br>A<br>N<br>D<br><br>A<br>N<br>A<br>L<br>Y<br>Z<br>E | <p><b><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u></b></p> <p><b><u>HOMEWORK:</u></b></p> <p>The cost of a sweater at your favorite store is \$45.00. During the “Day and Night Sale”, it is marked 30% off the original price. In addition, 10% will be taken off for using your credit card.</p> <p>The sales associate asks for your preference:</p> <p>A) Take 40% off the original price of the sweater?</p> <p style="text-align: center;">OR</p> <p>B) Take 30% off the original price of the sweater and then 10% off the remaining cost?</p> <p>Which option would you choose? Explain why.</p> | <p><b><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u></b></p> |

## Problems with Percents

Julie and her mother are shopping for some new jeans for school. They notice a rack of jeans with this sign on top of it:



Julie finds a pair of jeans on the rack, but unfortunately part of the price tag has been torn off. The tag looks like this:



Julie's mom claims that they can take 65% off the original price to determine the cost of the jeans. Julie claims that her mother is incorrect. Who is right – Julie or her mom? Explain your reasoning. What price will they pay for the jeans?

**Consider:**

- What would happen to the final price if the 40% discount was taken first and the 25% discount was taken second? Explain your thinking.
- What percent of the original price is the final price? Can you find a general rule for finding the sale price of an item that is discounted several times based on its previous price?

(Adapted from "Making Sense of Percents", *Mathematics Teaching in the Middle School*, September, 2003.)

**POSSIBLE SOLUTIONS:**

***Adding the two percents to find the discount – an incorrect solution:***

The following solution would be used by students who incorrectly think Julie's mom is correct.

$$40\% + 25\% = 65\%$$

$$\$50 (.65) = \$32.50 \text{ discount}$$

$$\$50 - \$32.50 = \$17.50$$

The cost of the jeans is \$17.50. So, Julie's mom is correct.

***Determining the first discount and sale price and using that result to determine the second discount and sale price:***

The following solution is probably the most common solution students will use.

Julie is correct. You would have to find the price of the jeans after they were reduced by 25%. The 40% discount would be on the reduced price, not the original price. Since the reduced price is less than the original price, the 40% discount on the reduced price would be less than a 40% discount on the original price.

$\$50 (.25) = \$12.50$  first discount

$\$50 - \$12.50 = \$37.50$  first sale price

$\$37.50 (.40) = \$15.00$  second discount

$\$37.50 - \$15.00 = \$22.50$  final price

The cost of the jeans is \$22.50.

***Determining the sale price without finding the discount:***

Students who use this method realize that the sale price of an item is a percent of the original price that is  $1 - \text{discount rate}$ . (i.e., A discount rate of 25% means the sale price would be 75% of the original price.)

Julie is correct. The price on the tag would be 75% of the original price of \$50. You would then find 60% of that price to find the final price. We could take  $(.75 \times \$50)$  to find the first discount and then .60 times that answer to find the final price. So, the final price would be .60  $(.75 \times \$50)$  or \$22.50. That is not the same as what Julie's mom is saying. She is saying the final price would be 35% of the original price, which is  $(.35 \times \$50)$  or \$17.50.

$\$50 (.75) = \$37.50$  first sale price

$\$37.50 (.60) = \$22.50$  final price

**Consider:**

- ***What would be the final price if the 40% discount was taken first and the 25% discount was taken second? Explain your thinking.***

The price of the jeans would not change. Since multiplication is commutative, it would not matter in which order the discounts were taken.

$$\$50 \times .40 = \$20 \text{ discount}$$

$$\$50 - \$20 = \$30 \text{ first sale price}$$

$$\$30 \times .25 = \$7.50 \text{ second discount}$$

$$\$30 - \$7.50 = \$22.50 \text{ final price}$$

This is the same as the original problem.

Likewise,

$$.60 \times \$50 = \$30 \text{ first sale price}$$

$$.75 \times \$30 = \$22.50 \text{ second sale price}$$

You can also see this by using formulas.

P = original price

A = percent of price (1 - first discount rate)

B = second percent of price (1 - second discount rate)

To find the first sale price, you would multiply the percent of the price times the original price. This gives us  $A \times P$  or  $AP$ . To find the final price, you would multiply the previous price,  $A \times P$  or  $AP$ , by the second percent. This gives us  $B \times A \times P$  or  $BAP$ . If we reverse the percents, we would get  $B \times P$  or  $BP$  for the first sale price and  $A \times B \times P$  or  $ABP$  for the second price. Since multiplication is commutative,  $BAP$  is equivalent to  $ABP$ . Therefore, the order in which the discounts are taken does not matter.)

**Consider (cont'd.):**

- ***What percent of the original price is the final price? Can you find a general rule for finding the sale price of an item that is discounted several times based on its previous price?***

The original price is \$50 and the sale price is \$22.50. What percent of \$50 is \$22.50?

$22.50/50$  is 45%. So, \$22.50 is 45% of \$50.

If we think of the problem as 60% of 75% of \$50, we can see that we would take  $.60 \times .75 \times 50$ . If we multiply  $.60 \times .75$ , we see that we get 45%. So, 60% of 75% of \$50 is the same as 45% of \$50. In general, we would multiply  $(1 - \text{first discount rate}) \times (1 - \text{second discount rate})$  to find the sale price of an item discounted several times based on the previous price.