

Quadratic Quandary

Algebra

Overview: In this task, students use quadratic functions that model the height of rockets above the ground after they have been launched to graph the relationship between time and height. Students use the graph to determine the amount of time the rocket stays in the air and then describe how to find and interpret the x-intercepts of any quadratic function.

Mathematics: To solve the task successfully, students must graph the relationship between time and height for two quadratic functions. Students must be able to interpret the graph in terms of the context of the problem in order to set appropriate scales for the x- and y-axes and to interpret the x-intercepts. In addition, since one equation is given in terms of time and feet and the other in time and meters, students will need to adjust and interpret the scales and graphs for each situation. Students must then generalize how to determine and interpret the x-intercepts for any quadratic function by using a graph.

Goals:

- Graph and interpret the graph of a quadratic relationship in terms of the context of a motion problem.
- Explain how to determine and interpret the x-intercepts for any quadratic function by using a graph.

Algebra Content Standards:

21.0 Graph quadratic functions and know that their roots are the x-intercepts.

Building on Prior Knowledge:

Materials: Quadratic Quandary (attached), calculators, graph paper, graphing calculators

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P	<u>HOW DO YOU SET UP THE LESSON?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.	<u>HOW DO YOU SET UP THE LESSON?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.
	<u>SETTING THE CONTEXT FOR THE TASK</u> Ask students to follow along as you read the problem: Two friends, Adam and Alyssa, are members of model rocket clubs at their schools. Each of their schools is having a competition to see whose model rocket can stay in the air the longest. The science teachers in each school have helped the students construct equations that describe the height of the rockets from the ground when they have been launched from the roofs of the schools. Following are Adam's and Alyssa's equations:	<u>SETTING THE CONTEXT FOR THE TASK</u> It is important that students have access to solving the problem from the beginning. • Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. • Make certain that students understand the vocabulary used in the problem. • Check on students' understanding of the task by asking several students what they know and what they are trying to find when solving the problem. • Be careful not to tell students how to solve the task, or to set up a procedure for solving the task, because your goal is for students to do the problem solving.
	Adam: $h = -16t^2 + 40t + 56$ where t is measured in seconds and h is measured in feet. Alyssa: $h = -5t^2 + 15t + 18$ where t is measured in seconds and h is measured in meters. 1. Use a graph to determine whose rocket stays in the air the longest. Explain how you used the graph to answer the question. 2. Explain how to find the x-intercepts of any quadratic function by graphing. In general, what do the x-intercepts of a quadratic function mean?	

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P S E T U P	<u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u> Remind students that they will be expected to: • justify their solutions in the context of the problem. • explain their thinking and reasoning to others. • make sense of other students' explanations. • ask questions of the teacher or other students when they do not understand. • use correct mathematical vocabulary, language, and symbols. Tell students that their groups will be expected to share their solutions with the whole group using the board, the overhead projector, etc.	<u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u> Setting up and reinforcing these expectations on a continual basis will result in them becoming a norm for the mathematics classroom. Eventually, students will incorporate these expectations into their habits of practice for the mathematics classroom.

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E X P L O R E E X P L O R E E X P L O R E	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> Circulate among the groups as students work privately on the problem. Allow students time to individually make sense of the problem. <u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to find? • What does each equation tell you? • What are the independent and dependent variables? <u>Possible misconceptions or errors:</u> • Setting appropriate scales for the graphs of each equation may pose a problem for some students. You might say, “How high do you think the rocket will go? How long do you think it will stay in the air? How is the second equation different from the first?” • Some students may not understand that to find how long the rocket stays in the air, you must determine when it hits the ground. You might say, “Think about the flight of the rocket. What would a graph of its height look like?” • Students who are using a graphing calculator may graph the equation without taking into consideration the context of the problem in terms of time and distance. You might point to the part of the parabola that is in the 2nd and 3rd quadrants and ask, “So, what does this part of the graph mean in terms of this problem?” • Students may attempt to graph both equations on the same set of axes. You might ask, “What would the labels for the axes of the first graph be? What about the second graph?”	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> It is important that students be given private think time to understand the problem for themselves and to begin to solve the problem in a way that makes sense to them. <u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method. <u>Possible misconceptions or errors:</u> It is important to have students explain their thinking before assuming they are making an error or have a misconception. After listening to their thinking, ask questions that will move them toward understanding their misconception or error.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <u>Possible Solution Paths</u> Strategies will be discussed as well as the questions that you might ask students. Representations of these solutions are included at the end of this document.	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <u>Possible Solution Paths</u> Questions should be asked based on where the learners are in their understanding of the concept. It is important that student responses are given both in terms of the context of the problem and in correct mathematical language. ** Indicates key mathematical ideas in terms of the goals of the lesson
	<i>Graphing by plotting points:</i> Ask questions such as: • What do you think the graph might look like? • What do the “t” and “h” represent in this problem?	<i>Graphing by plotting points:</i> Possible Student Responses • Depending on prior experience with quadratic functions, students should realize that a graph of time and height should go up and then down. • Students should state that the “t” represents the amount of time in seconds that the rocket is in the air and the “h” represents height of the rocket above the ground. ** The function describes a relationship between time and height for this problem, both of which have positive values only. • Students should realize that both “t” and “h” can have only positive values and that “t” should begin at 0.
	• What do you think would be sensible values for “h” and “t” for each equation? What might be a good value to begin with? • How will you know when you have found the solution to the problem?	• Students should state that the rocket would stay in the air for a set amount of time and will then come back and hit the ground. The time at which each graph intersects the x-axis will be the amount of time each was in the air. Whichever rocket hit the ground at a time after the other rocket, will be the one that stayed in the air the longest. ** The fact that the rocket will eventually hit the ground means that for some value of “t”, “h” will be 0. The value of “t” when “h” is 0 is the amount of time spent in the air.

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E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> Since the solution is not a whole number, you might need to prompt students to choose values that are not whole numbers. • How will you know when you are close to finding the solution? Can time be a number other than a whole number? • What mathematical term can we use to describe the solution? • How could you find the solution for any quadratic function by graphing? • How many solutions could a quadratic function have?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> • Students should recognize that if for 2 consecutive values of t, one value of h is positive and one is negative, then the solution is between those 2 values. • Students should be able to identify the solution as the x-intercept. ** An x-intercept is the value of the independent variable, in this case "t", at which the dependent variable, in this case "h", is 0. On a graph, this is the point at which the graph "crosses" or intersects the x-axis, or in this case, the t-axis. • The last 2 questions will be explored extensively in the Share, Discuss, and Analyze phase.

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EXPLORE EXPLORE EXPLORE	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Graphing using the graphing calculator:</i> Ask question such as: • What do you think the graph might look like? • What do the “t” and “h” represent in this problem? What do you think would be sensible values for “t” and “h”? • What would be a good window to use on the calculator? • How will you know when you have found the solution? • What mathematical term can we use to describe the solution? • How could you find the solution for any quadratic function by graphing? • How many solutions could a quadratic function have?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Graphing using the graphing calculator:</i> Possible Student Responses • Depending on prior experience with quadratic functions, students should realize that a graph of time and height should go up and then down. • Students should state that the “t” represents the amount of time in seconds that the rocket is in and air and the “h” represents height of the rocket above the ground. ** The function describes a relationship between time and height for this problem, both of which have positive values only. • Students should realize that both the x- and y-windows should have a minimum value of 0. • Students should state that the rocket will stay in the air for a set amount of time and will then come back and hit the ground. The time at which each graph intersects the x-axis will be the amount of time each was in the air. Whichever rocket hit the ground at a time after the other rocket, will be the one that stayed in the air the longest. ** The fact that the rocket will eventually hit the ground means that for some value of “t”, “h” will be 0. The value of “t” when “h” is 0 is the amount of time the rocket spent in the air. • Students should be able to identify this as the x-intercept. ** An x-intercept is the value of the independent variable, in this case “t”, at which the dependent variable, in this case “h”, is 0. On a graph, this is the point at which the graph “crosses” or intersects the x-axis, or in this case, the t-axis. • Students who have a fairly good knowledge of the graphing calculator may realize that they can use several features of the calculator to determine the solution, such as the “TRACE” features or the “zero” features. Ask students to explain WHY they chose that feature and what it means in the context of the problem.

E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Graphing using the graphing calculator:</i> • How could you find the solution for any quadratic function by graphing? • How many solutions could a quadratic function have?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Graphing using the graphing calculator:</i> Possible Student Responses • The last 2 questions will be explored extensively in the Share, Discuss, and Analyze phase. ** NOTE: Some students may think that the time and height of the rockets cannot be compared since they are in different units (time vs. ft. and time vs. m). Ask students to explain what each equation represents so that they realize, in both cases, they are looking for the amount of time the rocket stays in the air. In both cases, the unit of time is seconds so they can compare the two amounts.
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Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> <u>Possible Solutions to be Shared</u> You might begin by asking the students who plotted points to share their solutions. <i>Graphing by plotting points:</i> Ask questions such as: • What does each graph represent? What are the independent and dependent variables for each? • Describe how you constructed your graphs. How did you determine the scales for each graph?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> The purpose of the discussion is to assist the teacher in making certain that the goals of the lesson are achieved by students. Questions and discussions should focus on the important mathematics and processes that were identified for the lesson. ** Indicates key mathematical ideas in terms of the goals of the lesson <u>Possible Solutions to be Shared</u> If some students solved the problem by plotting points, ask them to share their solution(s) first, since they needed to estimate to determine the solution. Then share the solution using the graphing calculator. <i>Graphing by plotting points:</i> Possible Student Responses • Students should state that each graph represents a quadratic function relating time and height. In the first graph, height is in feet. In the second, it is in meters. ** The graph of a quadratic function produces a “parabola”. • Students should state that t, the independent variable, has values beginning at 0. Since h, the dependent variable, is in either feet or meters, its values start at 0 but go much higher than the time variable.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Graphing by plotting points (cont'd.):</i> • How did you know when you found the solution to the problem? • What mathematical name do we use to describe the solution? <i>Graphing using the graphing calculator:</i> If possible have students demonstrate their solution on the overhead calculator. Ask questions such as: • What are the independent and dependent variables for each of your graphs? • How did you know what values to use to set the window?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Graphing by plotting points (cont'd.):</i> • Students should describe that to determine how long each rocket stayed in the air, they had to approximate when each hit the ground. This would be the approximate value of t when the height had a value of "0". Whichever rocket had the highest value of t, would have stayed in the air the longest. Since Adam's rocket stayed in the air 3.5 sec. and Alyssa's stayed in the air over 3.9 sec., Alyssa's rocket stayed in the air the longest. • Students should say that the solution is the x-intercept. ** An x-intercept is the value of the independent variable, in this case "t", at which the dependent variable, in this case "h", is 0. On a graph, this is the point at which the graph "crosses" or intersects the x-axis, or in this case, the t-axis. Finding the solution is the same as finding the x-intercept. <i>Graphing using the graphing calculator:</i> Possible Student Responses • Students should state that each graph represents a quadratic function relating time and height. In the first graph, height is in feet. In the second, it is in meters. ** The graph of a quadratic function produces a "parabola". • Students should state that t, the independent variable, has values beginning at 0. Since h, the dependent variable, is in either feet or meters, its values start at 0 but go much higher than the time variable.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Graphing using the graphing calculator (cont'd.):</i> Ask questions such as: • How did you know when you found the solution to the problem? • What mathematical name do we use to describe the solution?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Graphing using the graphing calculator (cont'd.):</i> Possible Student Responses • Students may have found the solution several ways – using the “Trace” feature, the “Zero” feature, or using the “Table” function. (See POSSIBLE SOLUTIONS on attached pages.) Ask students to demonstrate each of these. • For each solution, students should describe that to determine how long each rocket stayed in the air, they had to find out when each hit the ground. This would be the value of t when the height had a value of “0”. Whichever rocket had the highest value of t would have stayed in the air the longest. Since Adam’s rocket stayed in the air 3.5 sec. and Alyssa’s stayed in the air over 3.9 sec., Alyssa’s rocket stayed in the air the longest. ** The solution for each graph is the point at which the y-, or dependent variable, value is 0. • Students should say that the solution is the x-intercept. ** An x-intercept is the value of the independent variable, in this case “t”, at which the dependent variable, in this case “h”, is 0. On a graph, this is the point at which the graph “crosses” or intersects the x-axis, or in this case, the t-axis. Finding the solution is the same as finding the x-intercept.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
SHARE DISCUSS AND ANALYZE	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Graphing using the graphing calculator (cont'd.):</i> <i>Question 2:</i> Pose the following questions to the entire class: If you were asked to find the solution of a quadratic function by graphing, how many solutions could you get? Draw examples of each. How would you find the solutions? If you were asked to find the solution of a quadratic function WITHOUT graphing, how might you do that? What is true about every solution or x-intercept? What is another name for the solution or x-intercept? So what does it mean to find the solution, find the x-intercepts, or find the roots of an equation? ASSIGNMENT:	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Graphing using the graphing calculator (cont'd.):</i> <i>Question 2:</i> Possible Student Responses - Give students time to think about this before asking for answers. Encourage them to draw rough sketches of the examples. Students should state that a quadratic function could have 0, 1, or 2 solutions. (See POSSIBLE SOLUTIONS on attached pages.) - Students should state that, whether using a graphing calculator or plotting points, they try to find the value(s) of x that result in a y value of 0. ** The solution of a quadratic function is the point at which the y, or dependent variable, value is 0. Possible Student Responses - Students should realize that finding the x-intercept on a graph means determining when the dependent value is 0. When given an equation rather than a graph, that would mean that the dependent variable is replaced by 0. ** The solution of a quadratic function is the point at which the y, or dependent variable, value is 0 so the dependent variable would be replaced by 0. - Students should state that another name for finding the solution or finding the x-intercept is finding the root of the equation. - Press students to indicate that all three terms mean the same thing – finding the value of the independent variable for which the dependent variable is zero. ** Finding the roots, x-intercepts, or solutions of an equation means to find the value of x that results in a 0 value for y.

Quadratic Quandary

Two friends, Adam and Alyssa, are members of model rocket clubs at their schools. Each of their schools is having a competition to see whose model rocket can stay in the air the longest. The science teachers in each school have helped the students construct equations that describe the height of the rocket from the ground when it has been launched from the roof of the school. Following are Adam's and Alyssa's equations:

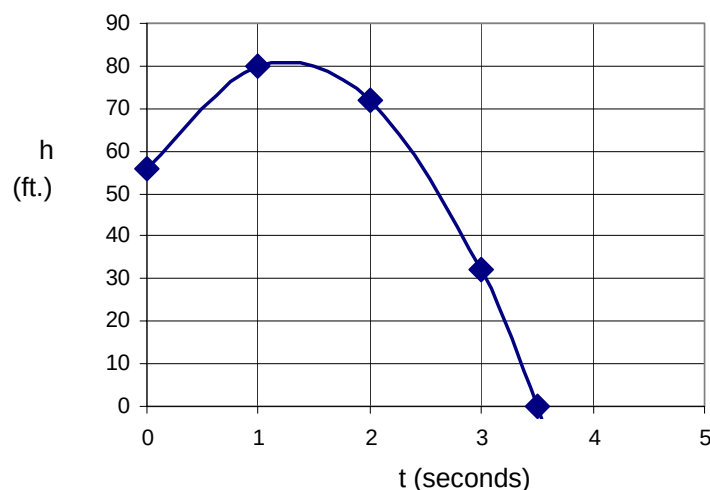
Adam: $h = -16t^2 + 40t + 56$ where t is measured in seconds and h is measured in feet.

Alyssa: $h = -5t^2 + 15t + 18$ where t is measured in seconds and h is measured in meters.

1. Use a graph to determine whose rocket stays in the air the longest. Explain how you used the graph to answer the question.
2. Explain how to find the x-intercepts of any quadratic function by graphing. In general, what do the x-intercepts of a quadratic function mean? How many x-intercepts can a quadratic function have?

POSSIBLE SOLUTIONS:*Graphing by plotting points:*Adam's Rocket: $h = -16t^2 + 40t + 56$

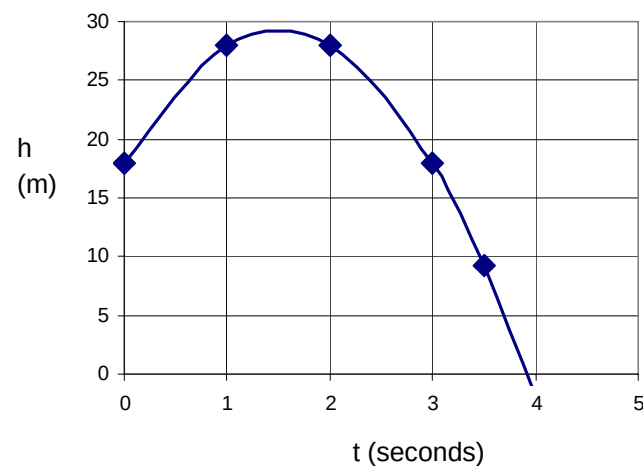
t	h
0	56
1	80
2	72
3	32
4	-40
	→ indicates the x-intercept is between 3 and 4 since 0 is between 32 and -40
3.2	20.16
3.4	7.04
3.6	-7.36
	→ indicates the x-intercept is between 3.4 and 3.6
3.5	0



At $t = 3.5$ seconds, $h = 0$. Adam's rocket stays in the air 3.5 seconds.

Alyssa's Rocket: $h = -5t^2 + 15t + 18$

t	h
0	18
1	28
2	28
3	18
4	-2
	→ indicates the x-intercept is between 3 and 4 since 0 is between 18 and -2
3.5	9.25
3.9	.45
3.95	-.7625
	→ indicates the x-intercept is between 3.9 and 3.95

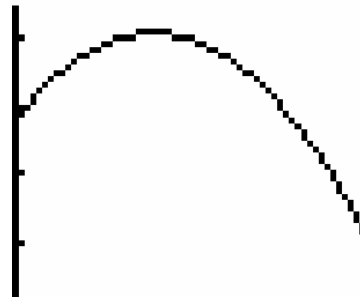


Between 3.9 and 3.95 seconds, $h = 0$. Alyssa's rocket stays in the air over 3.9 seconds. Alyssa's rocket stays in the air the longest.

Graph by using the graphing calculator:

Using the “TRACE” feature:

$$Y_1 = -16X^2 + 40X + 56$$

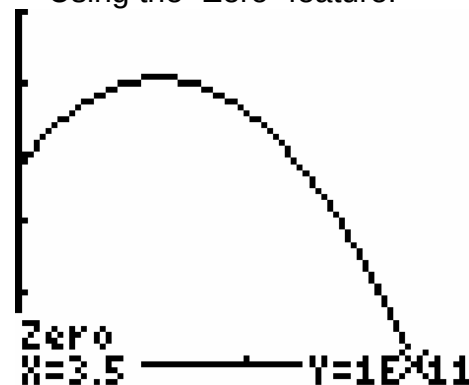


$$X=3.5106383 \quad Y=-.477682$$

- Press “Trace.”
- Move the cursor until the “Y” value is close to 0.
- You could also use the “Zoom” feature to get an answer closer to 0.

** Adam’s rocket stayed in the air approximately 3.5 sec. because the y-value is close to 0.

Using the “Zero” feature:



- Press “2nd” “Trace.”
- Choose “2: zero”.
- Choose a left bound at which y is positive and a right bound at which the y-value is negative.

** Adam’s rocket stayed in the air 3.5 sec. because the y-value is 0 at 3.5 sec. You may need to have a discussion concerning the meaning of “1E -11.” (i.e. 10^{-11} is .00000000001, close to 0)

Using the “Table” feature:

X	Y ₁
0	56
1	80
2	72
3	32
4	-40
5	-144
6	-280

$$X=4$$

- Press “2nd” “Graph.”
- Between 3 and 4, the y- value changes from positive to negative.

X	Y ₁
3.2	20.16
3.3	13.76
3.4	7.04
3.5	0
3.6	-7.36
3.7	-15.04
3.8	-23.04

$$X=3.5$$

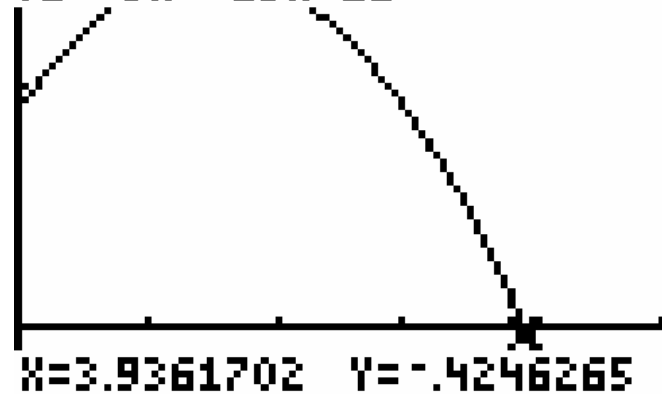
- Press “2nd” “Window.”
- TblStart 3.0
- $\Delta Tbl = .1$

** Adam’s rocket stayed in the air 3.5 sec. because the y-value is 0 at 3.5 sec.

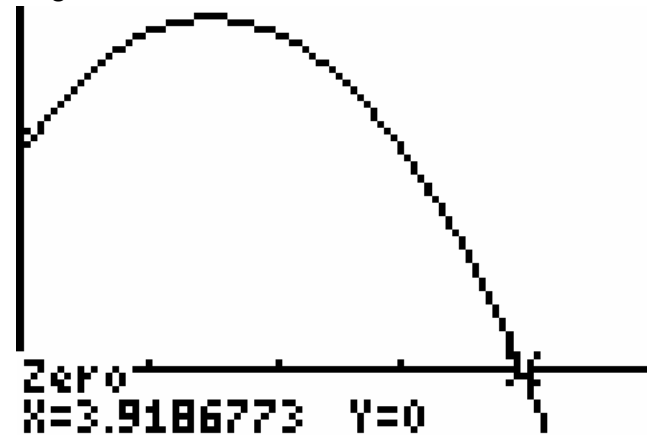
Alyssa's Rocket:

Using the "Trace" feature:

$$Y_2 = -5X^2 + 15X + 18$$



Using the "zero" feature:



Using the "Table" feature:

X	Y ₂	
0	18	
1	28	
2	28	
3	18	
3.9	-.42	
4	-32	
5	-72	

X=4

X	Y ₂	
3.7	5.05	
3.75	3.9375	
3.8	2.8	
3.85	1.6375	
3.9	.45	
3.95	-.7625	
4	-2	

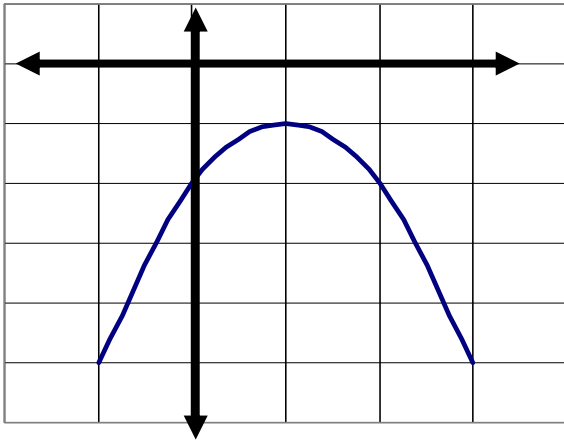
X=3.95

Alyssa's rocket stayed in the air for over 3.9 seconds.

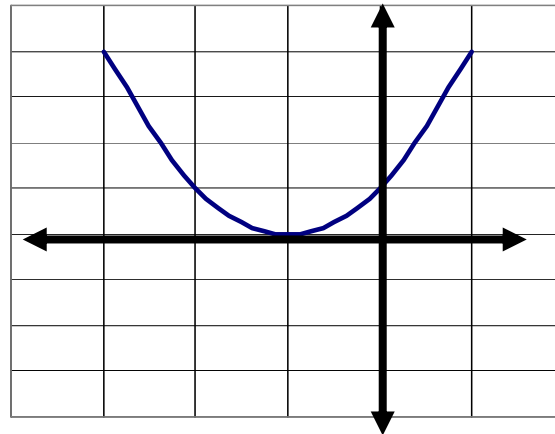
Question 2:

How many solutions can a quadratic function have?

0 solutions



1 solution



2 solutions

