

Ratios and Percents

7th Grade

Overview: In this lesson students relate ratios to percents.

Mathematics: To solve this task successfully, students must understand the ideas of ratio, scale factor, and multiplicative relationships.

Goals:

- Students will solve the three problems using a variety of strategies.
- Students will interpret scale factor in terms of the problem.
- Students will demonstrate their understanding of ratio.
- Students will justify their solutions to the problem.

Number Standards:

- 1.2 Add, subtract, multiply and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole number powers.

Building on Prior Knowledge:

- 1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

Materials: Ratios and Percents task (attached).

Phase	Action	Comments
S E T U P	Prior to the lesson: • arrange the desks so that students are in groups of 4. • determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. • place materials for the task at each grouping. • solve the task yourself.	Students will be more successful in this task if they understand what is expected in terms of group work and the final product. It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, look for the different strategies for solving the problem.
	HOW DO I SET-UP THE LESSON? Ask students to follow along as you read the problems. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem.	HOW DO I SET-UP THE LESSON? As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate that the goal is to determine the length and width of the rectangle described, the length of the string in the second problem, and the percent of butterfat in the resulting mixture. Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving.
E X P L O R E	<u>PRIVATE PROBLEM SOLVING TIME</u> Give students 5 - 7 minutes of private think time to begin to solve the problem individually. Circulate among the groups assessing students' understanding of the idea below. <u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> As you circulate among the groups, press students to think in terms of scale factors and ratios. You may ask them How would you know if the length and width are correct? What can you do to test your ideas? How would you model the string being cut? How would you represent the 25% longer piece of string? Will the resulting mixture have more or less fat than the 50 gallons of 20% butterfat? Will the resulting mixture have more or less fat than the 150 gallons of 4% butterfat? How could you test your ideas?	<u>PRIVATE PROBLEM SOLVING TIME</u> Make sure that the talking of other students does not interrupt students' thinking. If students begin talking, tell them that they will have time to share their thoughts in a few minutes. <u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> The teacher's role when students are working in small groups is to circulate and listen with the goal of understanding students' ideas and asking questions that will advance student work.

What do I do if students have difficulty getting started?

Allow students to work in their groups to solve the problem. Assist students/groups who are struggling to get started by prompting with questions such as:

- **How are the length and width related? How can you compare the different pieces of string? How will the mixture be different than the cream and milk? How do you know?**
- **Could you draw a diagram that shows the [rectangle] [string] [mixture]?**

What misconceptions might students have?

Look for and clarify any misconceptions students may have.

- a. *Students may not have a clear idea of ratio. **What does it mean that the ratio of length to width of the rectangle is 4 to 3? What would happen to the width if the length is doubled?***
- b. *Students may understand percent in relationship to the whole. **What does 25% longer than the first one mean? The second piece of string is 25% longer than what? The third piece of string is 25% shorter than what?***
- c. *Students may not understand that scale factors are dimensionless. **Although the area of the rectangle is given in square units, the scale factor determined by the ratio of 4 to 3 in the rectangle is dimensionless.***

What do I do if students have difficulty getting started?

By asking a question such as “**Can you state in your own words what the problem is asking you to do?**” the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine.

What misconceptions might students have?

Misconceptions are common. Students may have learned one way of solving this kind of problems. Some strategies for helping students gain a better understanding include:

- Ask students to try different representations of the same problem and find connections among them.

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will approach the problem using a variety of strategies. Some strategies are shown below. Questions for assessing understanding and advancing student learning are listed for each.

Question 1. *Using a table.*

How do you know which numbers to multiply by?

How do you know that you have found the answer?

What happens if you do not multiply the width and the length by the same number?

Question 1. *Using a diagram.*

How would you find the length and the width? What is the area of each smaller rectangle inside? Where in your picture is the 4 to 3 ratio represented?

Question 2. *Diagram.*

Could you show what each segment represents? Why did you make twelve divisions? Where in your diagram is 25% longer? Could you represent that using symbols? Could you write an expression that describes your picture?

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

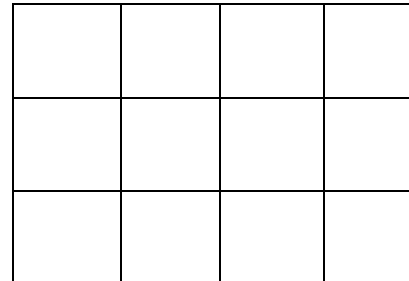
Question 1. *Using a table.*

Students could use a table with different widths and lengths to determine the width and length of the rectangle.

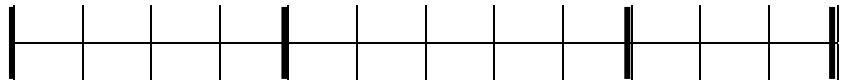
Width	Length	Area
$3 \times 2 = 6$	$4 \times 2 = 8$	48
$3 \times 3 = 9$	$4 \times 3 = 12$	108
$3 \times 4 = 12$	$4 \times 4 = 16$	192
$3 \times 5 = 15$	$4 \times 5 = 20$	300

Question 1. *Using a diagram.*

Students may draw a rectangle and rows and columns to represent the ratio.



Question 2. *Diagram.*



Question 3. *Using a diagram.*

How do you know that the mixture is 200 gallons? How can your diagram help you find out the percent of butterfat of the mixture? Is the percent of butterfat in the mixture closer to 4% or 25%? How do you know?

Question 3. *Using a diagram.*

50 gallons cream

[illegible]

200 gallon mixture

S H A R E, D I S C U S S, A N D A N A L Y Z E	FACILITATING THE GROUP DISCUSSION <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of their representations. The emphasis of the discussion should be on helping students understand the idea of ratio, percent, and making connections among their representations. Ask students to post their work in the front of the classroom. The goal is to discuss mathematical ideas associated with finding scale factors, using ratios to compare linear measurements, and to find percents. <i>What questions can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? (Driving Questions *)</i> Ask students to present their solutions and explain how their different representations help them find the answers to the questions. You might ask: How do you know your solution maintained the 4 to 3 ratio? How did you figure out what 25% more than the first one was? Where do you see the relationship of the three pieces of string represented on the drawing? Can you always find out what the resulting percent of butterfat will be in the mixture? How?	FACILITATING THE GROUP DISCUSSION <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group. <i>What questions can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind?</i> Driving questions (*) Driving Questions have been provided because they will help to stimulate student interest, maintain the focus of the discussion on the problem context, and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or problem solving strategies. <u>Accountable TalkSM</u> <u>Pointing to the drawing</u> Asking student to use their representations as evidence for their reasoning. <u>Repeating or Paraphrasing Ideas</u> Ask other students to put explanations given by their peers into their own words. This is a means of assessing understanding and providing others in the class with a second opportunity to hear the explanation.
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		<i>Position-Driven Discussion.</i> Press students to take a position and to support their claims with evidence from the drawings. Students must say why their drawings represent that fraction of the pan of brownies. In doing so, they will have to provide reasons for their claims.
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Ratios and Percents

Your task: Read the three problems below and use pictures, diagrams, words, numbers, and/or symbols to show how you solve each of them.

1. The ratio of the length of a certain rectangle to its width is 4 to 3. Its area is 300 square inches. What are its length and width?
2. A length of string that is 180 cm long is cut into 3 pieces. The second piece is 25% longer than the first, and the third piece is 25% shorter than the first. How long is each piece?
3. If 50 gallons of cream with 20% butterfat are mixed with 150 gallons of milk with 4% butterfat, what percent butterfat is the resulting mixture?

Make sure to explain how and why your method for solving each problem works.

Smith, M.S., Silver, E.A., and Stein, M.K. (2005). *Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathematics Teaching and Learning*. P. 58.