

Shrinking and Enlarging – 7th Grade

Overview: In this lesson students determine the scale factor to enlarge and reduce a poster on a copier machine.

Mathematics: To solve this task successfully, students must relate different scale factors as fractions, decimals, and percents to determine how to enlarge and reduce given drawings. The essence of the task is for students to understand the effect that taking a fraction of a fraction has as a scale factor (reducing a drawing to $\frac{1}{2}$ its size twice, reduces it to $\frac{1}{4}$ its original size) and translating that to a percent factor to be used in a copy machine. Students need to understand the concept of *scale factor* as the amount of enlargement or reduction needed to create an object similar in shape but different in size to the given object and that scale factors are positive numbers without units. Enlarging or reducing a picture means that the linear measurements of the picture are multiplied by a fixed number; the scale factor.

Goals:

- Students will solve the problem using a variety of strategies.
- Students will interpret scale factor in terms of the problem.
- Students will demonstrate the equivalence of rational numbers (fractions and percents) as scale factors.
- Students will justify their solutions to the problem.

Number Standards:

NS 1.2 Add, subtract, multiply and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole number powers.

Geometry Standards:

*** MG 1.2** Construct and read drawings and models made to scale.

Building on Prior Knowledge:

NS 1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

NS 1.6 Calculate the percentage of increases and decreases in quantity.

Materials: Making Copies task (attached); chart paper; graph paper; markers

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	Action	Comments
S E T U P	Prior to the lesson: • arrange the desks so that students are in groups of 4 • determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. • place materials for the task at each grouping. • solve the task yourself. <u>HOW DO I SET-UP THE LESSON?</u> Ask students to follow along as you read the problem. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem.	Students will be more successful in this task if they understand what is expected in terms of group work and the final product. It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different strategies for solving the problem will be given. <u>HOW DO I SET-UP THE LESSON?</u> As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate the goal is to determine the scale factor and demonstrate how to use it to enlarge and reduce an advertisement that is $8\frac{1}{2}$" by 11". Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving.
	<u>PRIVATE PROBLEM SOLVING TIME</u> Give students 5 - 7 minutes of private think time to begin to solve the problem individually. Circulate among the groups assessing students' understanding of the idea below. <u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> As you circulate among the groups, press students to think in terms of scale factors and ratios. You may ask them How would you know if the poster fits on the larger papers? What can you do to test your ideas? How would you find the scale factor Raphael needs to use to increase his poster?	<u>PRIVATE PROBLEM SOLVING TIME</u> Make sure that students' thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes. <u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> The teacher's role when students are working in small groups is to circulate and listen with the goal of understanding students' ideas and asking questions that will advance student thinking.
E X P L O R E		

What do I do if students have difficulty getting started?

Allow students to work in their groups to solve the problem. Assist students/groups who are struggling to get started by prompting with questions such as:

- **How can you compare the $8\frac{1}{2}$ " by 11" ad to the 13" by 22" newspaper page? What will change on the original poster? By how much? How do you know?**
- **Could you draw a diagram that shows the changes in dimensions from the smaller to the larger ad?**
- **For the full-page ad to be similar to Raphael's model, what must be true?**
- **Suppose you have a 3" by 5" index card. What would its dimensions be if it was enlarged by a scale factor of 1.5?**

What misconceptions might students have?

Look for and clarify any misconceptions students may have.

- a. *Students may not have a clear idea of similarity. **How will the enlarged/reduced picture be similar/different than the original?**(Similar objects have same shape but different sizes.) Some students may need to understand the difference between congruent and similar. (Congruent figures have the same shape AND same size (dimensions.) Congruent figures can be said to be similar because they have a scale factor of "1". But not all similar figures are congruent.*
- b. *Students may have learned that whenever you multiply you always get a larger number. **Think of a number between 1 and 10. Think of another number between 0 and 1. Multiply them. What do you get? Try multiplying two numbers between 0 and 1. What do you get? When will you get a larger number as a result of multiplying? (When multiplying by a number greater than 1).***
- c. *Students may not understand that scale factors are dimensionless. **Although the measures used to find the scale factors have units (inches, feet, etc.), the scale factor itself is dimensionless.***

What do I do if students have difficulty getting started?

By asking a question such as "**Can you state in your own words what the problem is asking you to do?**" the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine.

POSSIBLE STUDENT RESPONSES:

- The length and width of the original poster will both change. You have to multiply both the length and width by the same number.
- Students' drawings should indicate that the 2 rectangles have the "same shape."
- The dimensions of the full-page ad must both be a scale factor times the dimensions of the original ad. (i.e. Both dimensions of the full page ad are "x" times the dimensions of the original ad."
- It would be 4.5" by 7.5".

What misconceptions might students have?

Misconceptions are common. Students may not have a clear idea that similar figures have the same shape (i.e. triangles can only be similar to other triangles; rectangles can only be similar to other rectangles) but different sizes (i.e. the similar figure's dimensions are results of multiplying the original figure's dimensions by the SAME number. Some strategies for helping students gain a better understanding include:

- Ask students to use other representations of the same problem and find connections among them.
- Draw 2 pairs of figures, one pair being similar and the other pair not being similar (i.e. two rectangles – one being 3"by 5" and the other being 6" by 10"; two rectangles – one being 3" by 5" and the other being 4" by 10") to give students a visual representation of similar figures.

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will approach the problem using a variety of strategies. Some strategies are shown below. Questions for assessing understanding and advancing student learning are listed for each.

QUESTION 1

A.. –using actual papers of the sizes described.

-For the full-page ad to be similar to Raphael's, what must be true?

-How would you know that the enlarged poster is similar to your original?

-How would you find the scale factor needed to create the enlargement?

 B.. –drawing scale models

How do you know that your measures are correct?

How did you find the scale factors?

For the full-page ad to be similar to Raphael's, what must be true?

Can you show on your diagram what is being compared to what?

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

QUESTION 1

A.. –using actual papers of the sizes described.

Students could use direct measures to derive the relationship between and among the lengths of the two posters (the original and the enlarged one).

POSSIBLE RESPONSES:

- All sides of the enlarged poster must be results of multiplying both $8\frac{1}{2}$ and 11 by the same number.
- To find the scale factor students could:
 - a. divide 22" by 11" to get a scale factor of 2. But a scale factor of 2 would mean the other dimension would be $8\frac{1}{2} \times 2$ or 16" which is too large.
 - b. Divide 13" by $8\frac{1}{2}$ " to get a scale factor of 1.53. A scale factor of 1.53 would mean the other dimension would be 11×1.53 or 16.83".
 - c. The full page ad could be 13" by 16.83".
 - d. Another option would be to ask the newspaper to reconfigure the original ad to be 6.5" by 11" and then use a scale factor of 2 to get a full page ad that is 13" by 22".

 B.. –drawing scale models

Students may draw pictures to represent the original poster and the enlargement and show the relationship of the linear measurements on them. It may be useful for some students to use graph paper to draw the pictures that represent the enlarged/reduced posters.

----- C.. –symbolic representation -How did you get the dimensions for the full page ad? Do you think you have found the largest possible dimensions for the ad? -Could you draw a picture to show what your calculations represent? QUESTION 2 a. How might we determine possible scale factors for the different size papers? Do any of these scale factors give us a similar poster with the same dimensions as the different size papers? Explain. b. Which size paper would you use? What is largest scale factor you could use for this size paper? c. What percent is the same as $\frac{1}{4}$? Is that a value that the machine will accept? How might you get that value using the possible machine values?	POSSIBLE RESPONSES - Students should indicate that to find the scale factor they either multiplied the original dimensions by the same number to get the dimensions for the full page ad or divided the dimensions of the full-page ad to get a scale factor that would work. (See POSSIBLE RESPONSES in part A.) - The students should indicate on their diagrams that they are comparing the lengths of both figures with the width of both figures. ----- C.. –symbolic representation Some students may only use numbers and symbols to represent the scale factors and answer the questions. See POSSIBLE RESPONSES for part A for sample responses as to how they determined their dimensions. - Students should be able to show in a diagram that both the length and width of the new rectangle are the same scale factor times the dimensions of the original rectangle. QUESTION 2 POSSIBLE RESPONSES: a. Students might state that a scale factor can be determined by dividing a dimension of the paper by a dimensions of the ad. For the 11” by 14” paper you could divide 11 (poster width) by $8\frac{1}{2}$ (original ad width) to get a scale factor of 1.29. You could then either divide 14 (poster length) by 11 (original ad width) which yields a scale factor of 1.27 OR you could multiply 11 (from the original ad) by 1.29 which gives a dimension of 14.19” for the poster. (Similarly, you could use the poster length scale factor first and apply the same procedure to the poster widths.) So the 11” by 14” paper is not similar to the $8\frac{1}{2}$” by 11” paper. Similar results occur for the 11” by 17” paper which is not similar to the original size paper. b. One possible answer might be to use the 11” by 17” paper and a scale factor of 1.29 yielding a poster 11” by 14. 19” . c. Students should state that $\frac{1}{4}$ is the same as 25% which is not a
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d. How does $12\frac{1}{2}\%$ relate to other percents you might know that are on the machine? What fraction is the same as $12\frac{1}{2}\%$? e. Is there more than one way to reduce the drawing to 36% of its size using the possible machine values?	value accepted by the machine. They might indicate that $\frac{1}{2}$ times $\frac{1}{2}$ is $\frac{1}{4}$ so you could reduce the drawing to $\frac{1}{2}$ its size by using 50% on the machine. You could then put the reduced drawing in the machine and reduce it to $\frac{1}{2}$ its size by using 50% on the machine again. A similar argument could be made by recognizing 50% times 50% is 25%. ($.50 \times .50 = .25$) d. Students might recognize that $12\frac{1}{2}\%$ is half of 25% so they could follow the procedure in part c and do another reduction of 50% on the machine. (50% times 50% times 50% is $12\frac{1}{2}\%$. ($.50 \times .50 \times .50 = .125$) Students might also recognize $12\frac{1}{2}\%$ is the same as $\frac{1}{8}$ and that $\frac{1}{8}$ is the same as $\frac{1}{2}$ times $\frac{1}{4}$. e. After doing the previous two problems, most students will recognize that 36% is 60% times 60%. Other possible answers might include: - 50% times 72% - 50% times 80% times 90% - 120% times 60% times 50%
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S H A R E, D I S C U S S, A N D A N A L Y Z E	<u>FACILITATING THE GROUP DISCUSSION</u> <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of their representations. The emphasis of the discussion should be on helping students understand the idea of scale factor and making connections among their representations. Ask students to post their work in the front of the classroom. The goal is to discuss mathematical ideas associated with finding scale factors, using ratios to compare linear measurements, and taking a fraction of a fraction. <i>What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? (Driving Questions denoted by *)</i> Ask students to present their solutions and explain how their different representations help them find the answers to the questions. You might ask: QUESTION 1: *How do you know your enlargement of the poster is similar to the original? *What was your scale factor? How did you determine your scale factor? -Did anyone use a different scale factor? How did you determine your scale factor? -Where do you see the relationship of a scale factor represented on the drawings? QUESTION 2: *Were any of the machine paper sizes similar to the original paper size? Explain how you know. *How did you figure out how to enter the scale factor on the copy machine to reduce the poster to $\frac{1}{4}$ its size? *How did you reduce the drawing to $12\frac{1}{2}\%$; 60%? Did anyone do it a different way? Explain.	<u>FACILITATING THE GROUP DISCUSSION</u> <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group. For this particular problem it could be best to share solutions in the same order they were discussed in the EXPLORE phase (Question 1 A-C) <i>What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? Driving questions (denoted by *)</i> Driving Questions have been provided because they will help to stimulate student interest, maintain the focus of the discussion on the problem context, and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or problem solving strategies. POSSIBLE RESPONSES : See the POSSIBLE RESPONSES in the Explore Phase above. <u>Accountable Talk</u> SM <i>Pointing to the drawing</i> Asking student to use their representations as evidence for their reasoning. <i>Repeating or Paraphrasing Ideas</i> Ask other students to put explanations given by their peers into their own words. This is a means of assessing understanding and providing others in the class with a second opportunity to hear the explanation. <i>Position-Driven Discussion</i> Press students to take a position and to support their claims with evidence. In doing so, students will have to provide reasons for their claims.
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Shrinking and Enlarging: Making Copies

Your task: Read the situation below and use pictures, diagrams, words, numbers, and/or symbols to show how to determine the scale factors you would need to enlarge or shrink an $8\frac{1}{2}$ " by 11" ad.

1. Raphael is closing his bookstore. He wants to place a full-page advertisement in the newspaper to announce his going-out-of-business sale. A full-page ad is 13" by 22", which allows for a white border around the ad. Raphael used his computer to make an $8\frac{1}{2}$ " by 11" model of the advertisement, but he wants the newspaper ad department to enlarge it to full-page size. Is this possible? Explain your reasoning.
2. Raphael wants to make sale posters by enlarging his $8\frac{1}{2}$ " by 11" ad. He thinks big posters will get more attention, so he wants to enlarge his ad as much as possible.

The copy machines at the copy shop have cartridges for three paper sizes: $8\frac{1}{2}$ " by 11", 11" by 14", and 11" by 17". The machines allow users to enlarge or reduce documents by specifying a percent between 50% and 200%. For example, to enlarge a document by a scale factor of 1.5, a user would enter 150% of its current size.

- a. Can Raphael make a poster that is similar to his original ad on any of the three paper sizes—without having to trim off part of the paper? Why or why not?
- b. If you were Raphael, what paper size would you use to make a larger, similar poster on the copy machine? What scale factor would you use? How would you enter it into the copy machine?
- c. How would you use the copier described above to reduce a drawing to $\frac{1}{4}$ of its original size? Remember, the copy machines only accept values between 50% and 200%?
- d. How would you use the copy machine to reduce a drawing to $12\frac{1}{2}\%$ of its original size?
- e. How would you use the copy machine to reduce a drawing to 36% of its original size?