

Linear Function: A Stack of Cups – Algebra

Overview: In this lesson students describe a relationship between the number of cups and the height of the stack of cups.

Mathematics: To solve this task successfully, students need to understand that a linear relationship is defined by a rate of change that is constant. The essential characteristic of linear functions is that they have a *constant rate of change*. The *change* in quantity y is a *constant multiple* of the *change* in quantity x .

$$\frac{\text{change in } y}{\text{change in } x} = k$$

A linear function can be represented using a table, a formula, a graph, a diagram, or a verbal description. What is essential is identifying the constant rate of change in each representation. These representations vary in usefulness and so it is also important to be able to move fluently from any one of these representations to any other.

Goals:

- Students will solve the problem using a variety of strategies.
- Students will develop an understanding of linear functions.
- Students will demonstrate an understanding of constant rate of change.
- Students will graph the relationships of the number of cups and the height of stack of cups.
- Students will explain the meaning of slope in the relationship of the stack of cups.
- Students will justify their solutions to the problem.

Algebra Standards:

6.0 Students graph a linear equation and compute the x and y intercepts.

7.0 Students verify that a point lies on a line, given an equation of the line.

Building on Prior Knowledge:

Grade 7 AF 1.4 Use algebraic terminology (eg., variable, equation, term, coefficient, inequality, expression, constant) correctly.

AF 1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

AF 3.3 Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio (“rise over run”) is the slope of a graph.

AF 3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle).

Materials: Stack of cups (attached); chart paper; graph paper; markers

Phase	Action	Comments
S E T U P	Prior to the lesson: • arrange the desks so that students are in groups of 4. • determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. • place materials for the task at each grouping. • solve the task yourself.	Students will be more successful in this task if they understand what is expected in terms of group work and the final product. It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions about the problem will be given.
	HOW DO I SET-UP THE LESSON? Ask students to follow along as you read the situation presented in the Stack of Cup task. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem.	HOW DO I SET-UP THE LESSON? As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate the goal is to determine and demonstrate the relationship between the number of cups and the height of the stack of the cups. Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving.

E X P L O R E	<u>PRIVATE PROBLEM SOLVING TIME</u> Give students 5 – 7 minutes of private think time to begin to solve the problem individually. Circulate among the groups assessing students' understanding of the idea below.	<u>PRIVATE PROBLEM SOLVING TIME</u> Make sure that students' thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes.
	<u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> As you circulate among the groups, press students to think about the relationship of the size of each cup and the stack of cups. After explaining their initial thinking you might say: What is the relationship of the number of cups in a stack with the height of their stack? What happens to the stack as you add cups? What quantities are related? How?	<u>FACILITATING SMALL GROUP PROBLEM SOLVING</u> The teacher's role when students are working in small groups is to circulate and listen with the goal of understanding students' ideas and asking questions that will advance student work.

What do I do if students have difficulty getting started?

Allow students to work in their groups to solve the problem. Assist students/groups who are struggling to get started by prompting with questions such as:

- **What would you need to do first?**
- **How could you tell what the relationship between one cup and the stack is?**
- **What comparisons can you make?**
- **What have you tried? Have you tried fewer cups? Is there a pattern? If so, can you describe it?**
- **What can you do to test different cases?**

What misconceptions might students have?

Look for and clarify any misconceptions students may have.

- a. *Not understanding the constant rate of change. **What is changing and what remains the same?***
- b. *Not understanding dependent/independent variables. **Remind them that y represents the quantity that changes according to changes in x.***
- c. *Formulas are effective recipes for producing the dependent variable for given values of the independent variable. **How is y related to x? What happens to y when x increases? What happens to y when you use different values for x?***

Which problem-solving strategies might be used by students?

How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will represent the relationship between the number of cups and the stack with a formula and a graph. Questions for assessing understanding and advancing student learning are listed for each.

The formula

What do I do if students have difficulty getting started?

By asking a question such as “**What is the problem asking me to do? Have I solved a similar problem before?**” the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine.

What misconceptions might students have?

Misconceptions are common.

Some strategies for helping students gain a better understanding include:

- Ask students to look for connections among the representations (the formula and the graph) of the problem.

POSSIBLE RESPONSES:

- a. Students might say that the height of a cup remains constant but the height of the stack changes as you add more cups.
- b. Students should realize that the height of the stack changes according to the number of cups.
- c. Students should understand that y increases as x increases.

Which problem-solving strategies might be used by students?

How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

POSSIBLE RESPONSES

a.	no. of cups	height of stack
	1	4 cm
	2	4.5 cm
	3	5 cm
	4	5.5 cm
	5	6 cm
	6	6.5 cm

	What is x? What is y? How can you see the rate of change in the formula? Does it make sense to let the value for x be negative? Why or why not? What happens if you input 0 in the formula? Is the function increasing or decreasing? How can you tell from the formula? What is the rate of change in this situation? ----- <u>The graph</u> How can you see the rate of change in the graph? At what point does the line cross the vertical axis? What do the coordinates of this point mean in the context of the Stack of Cups? Is the function increasing or decreasing? How can you tell from the graph?	<u>The formula</u> The formula is $y = .5x + 3.5$ where x represents the number of cups in the stack; y represents the height. Students might derive this formula by noticing in the chart that there is a difference of .5 between every two entries of height for which there is a difference of one between every two entries of “no. of cups.” The rate of change is the coefficient of x. In this case it is .5 which means the height of the stack increases .5 cm for every cup added. This is represented by the change of .5 from one cup to the next in the chart. It would not make sense for x to be negative since you cannot have a negative number of cups in the stack. If you put 0 in the formula, you get a value of 3.5. This would mean that for 0 cups the stack would be 3.5 cm high. Although this does not make sense for this problem, the 3.5 is the y-intercept of the equation. The function is increasing since the coefficient of x is positive. The rate of change, .5, is the amount the height increases for every cup added. <u>The graph</u>
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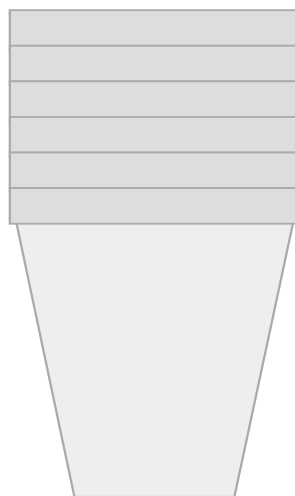
		Height of Stack of Cups <table><thead><tr><th>number of cups</th><th>Height in cm</th></tr></thead><tbody><tr><td>1</td><td>4.0</td></tr><tr><td>2</td><td>4.5</td></tr><tr><td>3</td><td>5.0</td></tr><tr><td>4</td><td>5.5</td></tr><tr><td>5</td><td>6.0</td></tr></tbody></table> The rate of change can be seen by the amount of change vertically from one point to the next for every unit. In this case, the height increases .5 cm for every 1 cup resulting in a rate of change of .5 cm per cup. The line would cross the y-axis at 3.5 cm. The coordinates of this point, the y-intercept, would mean that the height of the stack would be 3.5 cm for 0 cups. The function is increasing because the line is rising from left to right. As the number of cups increases, so does the height of the stack.	number of cups	Height in cm	1	4.0	2	4.5	3	5.0	4	5.5	5	6.0
number of cups	Height in cm													
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2	4.5													
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5	6.0													
S H A R E, D I S C U S,	FACILITATING THE GROUP DISCUSSION <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of their representations. Ask students to post their work in the front of the classroom. <i>What question can I ask throughout the discussion that will help</i>	FACILITATING THE GROUP DISCUSSION <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group. <i>What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? Driving questions (*)</i> Driving Questions have been provided because they will help to stimulate student interest, maintain the focus of the discussion on the problem context, and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or												

A N D A N A L Y Z E	<i>students keep the context and the goal of the problem in mind?</i> (Driving Questions *) How did you use the picture to decide what the relationship is between the number of cups in a stack and the height of the stack? Was the picture helpful in seeing this relationship? *What remained constant? What is the constant rate of change? *Describe the shape of your graph. Why is it a straight line? What would the line look like if this were a decreasing function?	problem solving strategies. <u>Accountable Talk</u> <i>Pointing to the graph</i> Asking student to point to the graph and showing where the evidence for their reasoning is. <i>Repeating or Paraphrasing Ideas</i> Ask other students to put explanations given by their peers into their own words. This is a means of assessing understanding and providing others in the class with a second opportunity to hear the explanation. <i>Position-Driven Discussion</i> Press students to take a position and to support their claims with evidence from the drawings. Students must say why their drawings represent the growth in height of the stack of cups. In doing so they will have to provide reasons for their claims.
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A stack of cups

Your task is to represent the relationship between the number of cups and the height of the stack of cups using a formula and a graph. Then answer the questions below about the stack of cups.

The paper cups shown below are identical and drawn full size.



- a) By making measurements, represent the relationship between the number of cups and the height of the stack using a formula and a graph.
- b) The graph can be drawn as a set of discrete points on a coordinate system. These points *lie along a line* and can be connected by a straight line.
 - ❑ Why is the line straight?
 - ❑ What are the slope and intercept of this line?
 - ❑ Interpret the meaning of the slope and intercept with regard to the number and size of the cups, or parts of the cups