

Two Storage Tanks Problem

Algebra 1

Overview: In this lesson, students determine when two tanks, one of which is leaking at a constant rate and one of which is being filled at a constant rate, contain the same amount of water. The situation is represented by graphs of their water levels over time.

Task: Storage Tank Problem (from NAEP Released Task Bank)

Goals:

- Students will solve the problem using a variety of strategies and representations.
- Students will develop an understanding of how to solve simultaneous linear equations.
- Students will develop an understanding of the meaning of solving simultaneous linear equations.
- Students will demonstrate an understanding of the meaning of the point of intersection of two lines.
- Students will explain and justify their solutions to the problem.

Content Standards:

9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically.

Building on Prior Knowledge: Seventh Grade Standards

- **AF 1.5**
- **AF 3.0**
- **AF 3.3**

To solve this task successfully, students need to understand that a linear function is defined by a rate of change that is constant. The function can be represented by a table, a formula, a graph, a diagram, or in words. The rate of change can be determined by examining features of each representation. (E.g., In a table, the rate of change is the ratio of the amount the dependent variable changes in relation to the corresponding changes in the independent variable. In a formula, the rate of change is the coefficient of x when the equation is written in the form $y = mx + b$. In a graph, the rate of change is the amount the y -coordinate changes from one point to another in relation to the amount x -coordinate changes. In words, the rate of change is indicated by identifying the independent and dependent variables and determining how values of the dependent variables change in relation to how the independent variable values change.)

Materials: Two Storage Tanks Tasks (attached); graph paper; graphing calculator (optional); ruler

- Determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together.
- Arrange the desks so that students are in groups of 3 or 4. Place materials at each grouping.

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P	<u>HOW DO YOU SET UP THE LESSON?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.	<u>HOW DO YOU SET UP THE LESSON?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.
	<u>SETTING THE CONTEXT FOR THE TASK</u> Ask students to follow along as you read the problem. Two large storage tanks, T and W, contain water. T starts losing water at the same time additional water starts flowing into W. The graph shows the amount of water in each tank over a period of hours. Assume that the rates of water loss and water gain continue as shown. At what number of hours will the amount of water in T be equal to the amount of water in W? Show or explain how you found your answer.	<u>SETTING THE CONTEXT FOR THE TASK</u> It is important that students have access to solving the problem from the beginning. Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. Check on students' understanding of the task by asking several students what they are trying to find when solving the problem.
	<u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u> Remind students that they will be expected to: - justify their solutions in the context of the problem. - explain their thinking and reasoning to others. - make sense of other students' explanations. - ask questions of the teacher or other students when they do not understand. - use correct mathematical vocabulary, language, and symbols.	<u>SETTING UP THE EXPECTATIONS FOR DOING THE TASK</u> Setting up and reinforcing these expectations on a continual basis will result in them becoming a norm for the mathematics classroom. Eventually, students will incorporate these expectations into their habits of practice for the mathematics classroom.

	Tell students that their groups will be expected to share their solutions with the whole group using chart paper, the overhead projector, etc.	
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E X P L O R E	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> Circulate among the groups as students work privately on the problem. Allow students time to individually make sense of the problem. <u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to find? • What can you tell me about Tank T? Tank W? • How will you know when the tanks contain the same amount of water? <u>Possible Solution Paths</u> Strategies will be discussed as well as the questions that you might ask students and the misconceptions that you might encounter. Representations of these solutions are included at the end of this document.	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> It is important that students be given private think time to understand the problem for themselves and to begin to solve the problem in a way that makes sense to them. <u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method. <u>Possible Solution Paths</u> Questions should be asked based on where the learners are in their understanding of the concept. It is important that student responses are given both in terms of the context of the problem and in correct mathematical language.
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Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Extending the lines until they intersect by drawing or using a ruler:</i>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Extending the lines until they intersect by drawing or using a ruler:</i>
		<u>Possible misconceptions or errors:</u> Students may extend the lines “freehand” without taking the relative rates of change into account. Share another solution in which the lines were drawn freehand that is different from the student’s solution.
	Ask questions such as: • Why did you extend your lines? • How do you know that we can extend them? • How can you be certain your lines are exact? • What might help you make certain the lines are exact? • How can you determine the rates of water loss and water gain? • How might you find the solution without using the graph?	Possible Student Responses • Students should state that they are trying to find where the two lines intersect. • Students should state that determining the rates of change for each line will help make the lines exact. • Students should mention that they know that the water continues to flow in or out at the same rate. • Students should state that the rates of water loss and gain can be determined by finding how much the water level changes relative to how much the time changes (rise/run; change in y/change in x). • Once students have solved the problem using the graph, press them to use the graphs of the lines to write equations for the water levels in each tank.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Extending the lines until they intersect by first determining the rates of change:</i> Ask questions such as: • Why did you extend your lines? • How did you determine the rates of water loss and water gain? • How did you know where to extend the lines? • How will you know when you have found the solution to the problem? • How might you find the solution without using the graph?	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Extending the lines until they intersect by first determining the rates of change:</i> Possible Student Responses • Students should state that they are trying to find where the two lines intersect. • Students should state that the rates of water loss and gain can be determined by finding how much the water level changes relative to how much the time changes (rise/run; change in y/change in x). • Students should state that they used the rates of change for each line to find additional points on each line. • Students should state that the solution will be where the two lines intersect. • Once students have solved the problem using the graph, press them to use the graphs of the lines to write equations for the water levels in each tank.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Making a table:</i>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Making a table:</i> <u>Possible misconceptions or errors:</u> Students may confuse the independent and dependent variables and place the water level values in the first column. Ask students to identify the independent and dependent variables in this problem. Students might try to find the coordinates of points in between grid lines, and thus their estimates might not be useful.
E X P L O R E	Ask questions such as: • How did you get the values for your table? • How can you determine the rates of water loss and water gain from your table? • How will you know when you have found the solution? • How might you find the solution without using the table?	Possible Student Responses • Students should state they identified points on each of the lines to get the values. • Students should state that they can find the rates of change by finding the ratio of the difference between two water level values to the difference between the two relative hour values. • Students should state that they are finding when both tanks have the same water level at the same time. • Once students have solved the problem using the table, press them to use their tables to write equations for the water levels in each tank. • Students may look for patterns within the tables and then continue the patterns until they find the point of intersection.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
EXPLORE	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Writing an equation:</i>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Writing an equation:</i> <u>Possible misconceptions or errors:</u> Students may confuse the y-intercept and the slope. If they are using the form “$y = mx + b$”, ask them to identify what “m” and “b” represent in general, and then what they would represent in this problem.
	Ask questions such as: • How did you know how to write an equation? • What does each part of your equation represent in terms of the problem? • What is the problem asking you to find? • How might you use the two equations to solve the problem?	Possible Student Responses • Students should state that the graphs are lines and that equations of lines are of the form “$y = mx + b$.” • Students should state that “m” represents the rate of change of the water level and that “b” represents the water level at 0 hours. • Students should state that they are trying to find when the two tanks have the same water level. • Students might state that the two equations would be “equal” when the water levels are the same. Determining the solution algebraically (using the equations) will be part of the whole-group discussion.

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E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Using a graphing calculator (optional):</i>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u> <i>Using a graphing calculator (optional):</i> In order to represent the problem on a graphing calculator, students would either put data points into a list (make a table) and then find the equation using linear regression, or write the equation in "Y = ". They would then graph the equations and either trace to find the solution or use the "point of intersection" feature on the calculator. Or, they might use the "table" feature of the calculator to find the point of intersection. Note: The trace feature does not always produce the exact solution.
	<i>Ask questions such as:</i> • How did you get the values for your lists? How do you see the rates at which the water level is changing in your lists?	<u>Possible misconceptions or errors:</u> In addition to the misconceptions noted for the other strategies, students may have problems seeing the graph if they have not set the window appropriately. You might ask them to look at the graph presented in the problem to help them determine a better window.
	<i>Ask questions such as:</i> • How did you get the values for your lists? How do you see the rates at which the water level is changing in your lists? • What does each part of your equation represent in terms of the problem? How do you see the rates at which the water level is changing in your equations? • How do you see the rates at which the water level is changing in your graph? In the table? • What is the problem asking you to find? • How would you see that in the graph? • How would you see that in the table? • How might you use the two equations to solve the problem?	Possible Student Responses Student responses should be similar to those noted for the other strategies.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> What solution paths will be shared and in what order and why? Most students will probably use the graph given with the problem and extend the lines. <u>Possible Solutions to be Shared</u> <i>Extending the lines until they intersect by simply drawing or using a ruler:</i> It might be interesting to begin with a graph in which the rates of change were not considered when extending the lines. If you are concerned that this might embarrass a student, you might have this solution prepared ahead of time and say, “When I did this problem with another class, one of the groups produced this solution”. Ask questions such as: • What were we trying to find in this problem? How would we see that in a graph? • What does the point of intersection mean in this problem? • In this solution, the point of intersection is at 7 hours. How does that relate to what you got? Why is it different? • How might we make the graph more exact?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> What solution paths will be shared and in what order and why? The order in which solutions are shared can assist the teacher in making certain the goals of the lesson are achieved. Starting with a solution path used by most or many of the students allows students access to the discussion. <u>Possible Solutions to be Shared</u> <i>Extending the lines until they intersect by simply drawing or using a ruler:</i> Starting with a solution that contains errors or misconceptions allows the misconceptions or errors to be addressed publicly in case other students or groups had the same misconception or error. <u>Possible Student Responses</u> • Students should state that we were trying to find when the two tanks had the same water level. In a graph, we would see that by find the point of intersection. • Students should state that the point of intersection in this problem is when both tanks have the same water level at the same time. • Students should state that this is not the same solution they got because the lines were drawn freehand so it is not an exact solution. • Students should state that finding the rates of change for each line would allow you to make a more exact graph.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Extending the lines until they intersect by first determining the rates of change:</i>	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Extending the lines until they intersect by first determining the rates of change:</i> This solution follows directly from the previous solution and allows connections to easily be made.
	D I S C U S S Ask questions such as: • At what rate is the water level changing in each tank? How do we see that in this graph? What mathematical word do we use? • How does that allow us to extend the graph? • How do we find the solution? What is the solution? • What does the solution mean in this problem? • Is there another way to find the solution? (The point of this question is to move students to eventually use equations to find the solution.)	Possible Student Responses • Students should state that Tank T's water level is decreasing at 50 gallons per hour, while Tank W's water level is increasing at 25 gallons per hour. Students should state that the rates of water loss and gain can be determined by finding how much the water level changes relative to how much the time changes. The mathematical word to describe this is 'slope'. • Students should state that they could now use rise/run or change in y/change in x to locate additional points. • Students should state that you continue until the two lines intersect at 8 hours when the water level is 500 gallons for both tanks. • Students should state that both tanks contain 500 gallons of water at 8 hours.
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Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Making a table:</i> Ask questions such as: • How did you get the values for your table? • How can you determine the rates of water loss and water gain from your table? What are the rates? • How will you know when you have found the solution? • How might you find the solution without using the table? (The point of this question is to move students to use equations to find the solution.)	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Making a table:</i> This strategy may, or may not, be appropriate to share at this time. If any students used a table to solve the problem, it will make a nice connection between the graphs and the equations. If students did not make tables but are familiar with how to write the equation of a line, then you might want to move to the next strategy. Possible Student Responses • Students should state they identified points on each of the lines to get the values for the table. • Students should state that they could find the rates of change by finding the ratio of the difference between two water level values to the difference between the two relative hour values. Tank T's water level is decreasing at 50 gallons per hour, while Tank W's water level is increasing at 25 gallons per hour. The mathematical word to describe this is 'slope'. • They should state that the solution occurs when both tanks have the same water level at the same time.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Writing an equation:</i> Ask questions such as: • How could we write an equation to describe the water levels of Tanks T and W? What is the equation? • What does each part of your equation represent in terms of the problem? • What is the problem asking you to find? • We got a solution of “500 gallons at 8 hours” using the methods above. How can we check that answer in our equations?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Writing an equation:</i> This strategy is easily connected to both the graphing and table strategies and addresses the main goal of the lesson. Possible Student Responses • Students should state that the graphs are lines and that equations of lines are of the form “$y = mx + b$.” The equation for Tank T is “$y = -50x + 900$.” The equation for Tank W is “$y = 25x + 300$.” • Students should state that “m” represents the rate of change of the water level (slope) and that “b” represents the water level at 0 hours (y-intercept). From the discussion above, students should state why they used “-50” and “25” for “m”. Students should also state that they determined the y-intercepts by looking at the graph and noting the location of each line at 0 hours. • Students should state that they are trying to find when the two tanks have the same water level. • Students should state that you could substitute 8 in for x and 500 in for y to see if it resulted in a correct solution. (Tank T: $500 = -50(8) + 900$ and Tank W: $500 = 25(8) + 300$)

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Ask questions such as (cont'd.): • How might you use the two equations to solve the problem? • How did we solve the problem using a graph or table? • So, what must be true about the two equations when their lines intersect? • How could we represent that algebraically? Since Tank T's equation is $y = -50x + 900$ and Tank W's equation is $y = 25x + 300$ and they have equal values when they intersect, we can replace the "y" in Tank W's equation with the "y" from Tank T's equation and solve for x. $-50x + 900 = 25x + 300$ $-75x = -600$ $x = 8$ If $x = 8$ then for Tank T, $y = -50(8) + 900$ so $y = 500$ and for Tank W, $y = 25(8) + 300$ so $y = 500$. • How does this compare to the answer we got using other strategies? • In general, how might we find the point of intersection of two lines using their equations? • Why does this work?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> Possible Student Responses (cont'd.) The purpose of these questions is to move students toward a method of solving simultaneous equations. • When using the graph or table, the x and y values were the same for both tanks. • Students should state that the two equations are "equal" or have equal x and y values when they intersect. • Students should state that this is the same answer they got using the other strategies. • Students should state that in general, you can find the intersection point for two lines by setting them equal to each other and solving for x. When x is substituted back in each of the equations, you should get the same y-value. • This works because at the point of intersection, both lines have the same x and y values. Since the y-values are the same, you can set the equations equal to each other to find the x value that makes them equivalent. Once you find the x-value, you can solve each equation for the y-value, which should be the same for both equations.

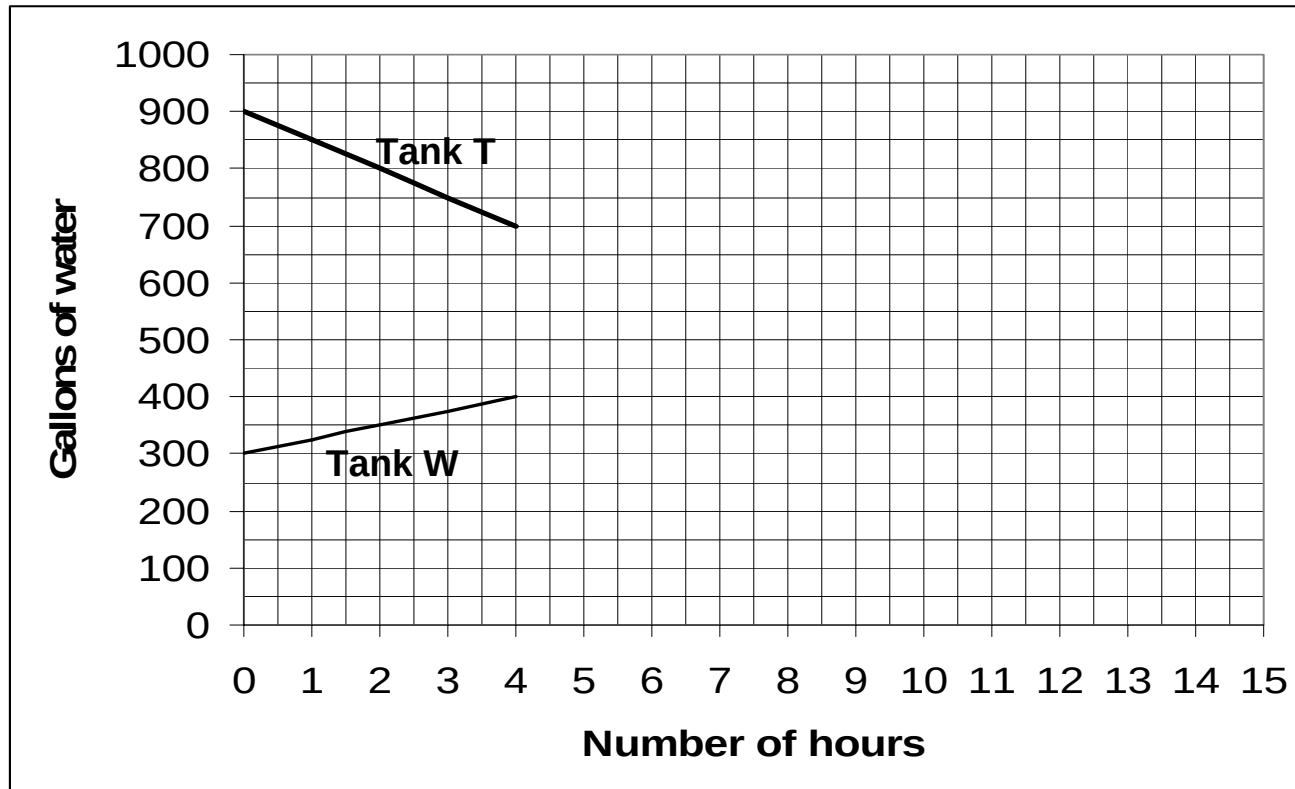
HOMEWORK:

You and your sister are beginning to save money from your allowances. You currently have \$25 and plan to save \$3.50 per week. Your sister currently has \$40 and plans to save \$2.25 each week. After how many weeks will you and your sister have the same amount of money? Explain how you solved the problem and why you chose that particular strategy.

*Adapted from **Algebra 1: Concepts and Skills**, McDougal Littell*

Two Storage Tanks

Two large storage tanks, T and W, contain water. T starts losing water at the same time additional water starts flowing into W. The graph below shows the amount of water in each tank over a period of time.

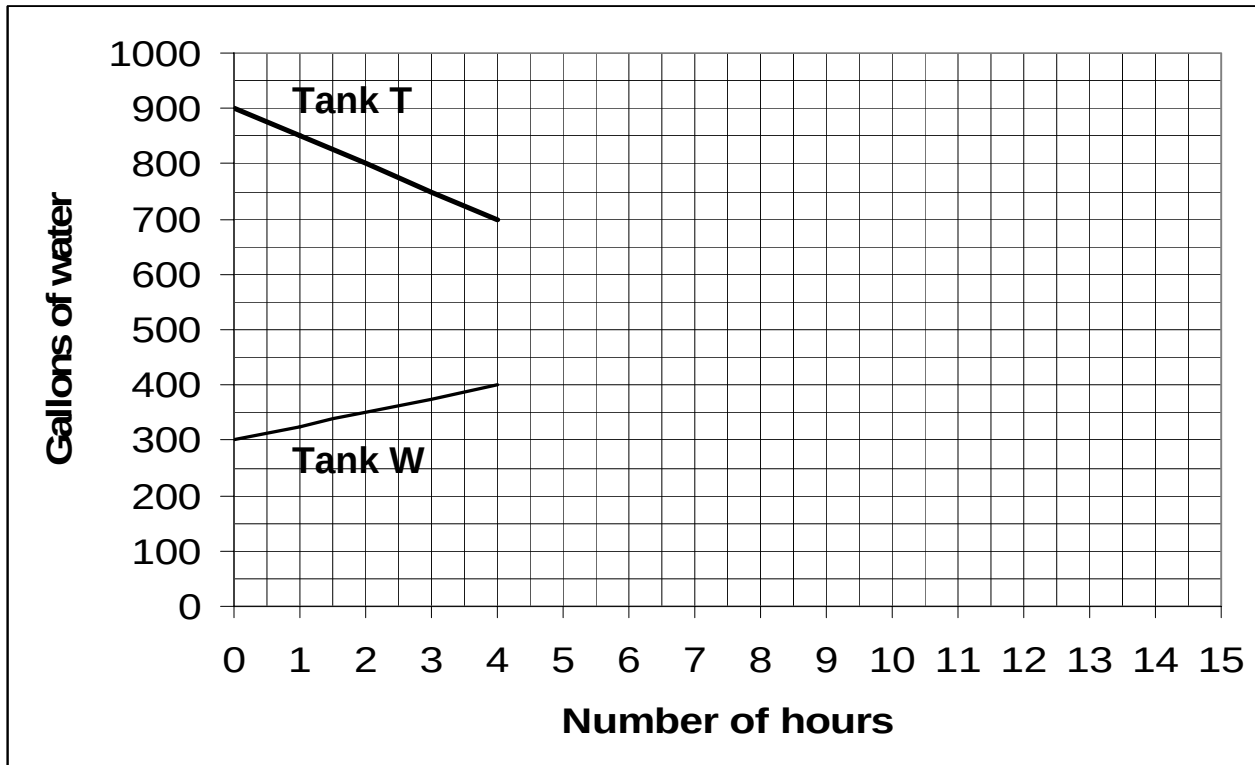


Assume that the rates of water loss and water gain continue as shown. At what number of hours will the amount of water in T be equal to the amount of water in W? Show or explain how you found your answer.

(From NAEP Released Items, 2003-8M10 No.13)

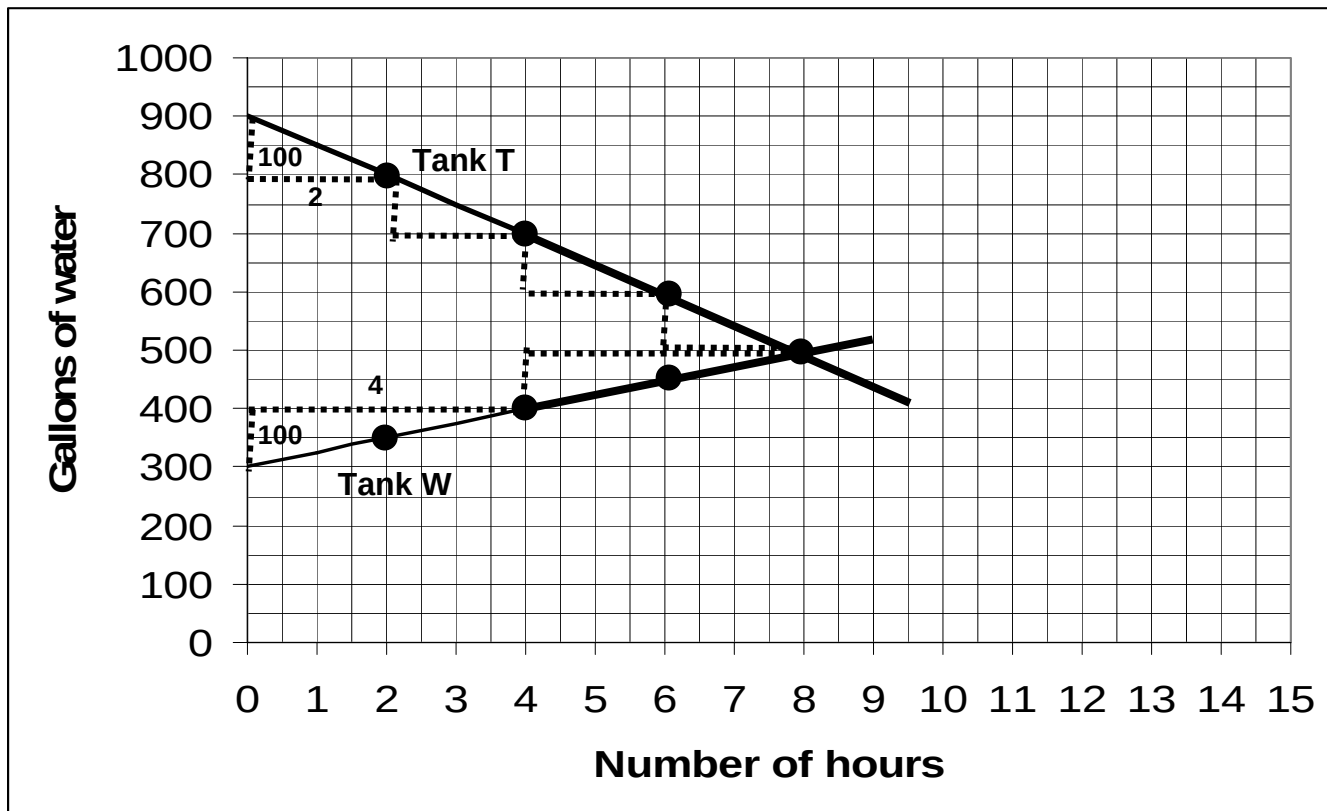
POSSIBLE SOLUTIONS:

Extending the lines until they intersect by simply drawing or using a ruler:



They have the same amount at 7 hours. I found out when the 2 lines crossed each other.

Extending the lines until they intersect by first determining the rate of change:



First, I found the rise over run for each line by finding how much the water level was changing every 2 hours. Since the water level in both tanks is described by a line, the rise/run would be the same everywhere on the lines. I used that to find other points on the lines and I extended each line until they intersected. They both have 500 gal. at 8 hrs. since both lines go through the point (8, 100).

Making a table:

HOURS	TANK T	TANK W
0 } 2	900 } -100	300 } 50
2 } 2	800 } -100	350 } 50
4 } 2	700 } -100	400 } 50
6 } 2	600 } -100	450 } 50
8 } 2	500 } -100	500 } 50

Tank T: $-100 \text{ gal./2 hr.} = -50 \text{ gal./hr.}$

Tank W: $50 \text{ gal./2 hr.} = 25 \text{ gal./hr.}$

I made a table and found how much water was in each tank at 0 and 2 hours. Then I found how much each tank was changing every 2 hours. Since we started with a line, the changes would be constant every 2 hours. So I kept adding the amount the tanks were changing until both tanks had the same amount of water at the same time. Both tanks contain 500 gallons at 8 hours.

Writing an equation:

Tank T: $y = \underline{-50}x + 900$

slope

↗ y-intercept

If you look at the graph, Tank T has a y-intercept (crosses the y-axis) at 900 and a slope of -50 since the water level goes down 50 gal. every hour. Then I used $y = mx + b$ to write the equation.

Tank W: $y = \underline{25}x + 300$

slope

↗ y-intercept

If you look at the graph, Tank W has a y-intercept (crosses the y-axis) at 300 and a slope of 25 since the water level goes up 25 gal. every hour. Then I used $y = mx + b$ to write the equation.

Since we want to know when the two tanks have the same amount of water, that means the amounts of water would be the same so the y values would be the same. So we can set the equations equal to each other:

$$-50x + 900 = 25x + 300$$

$$-75x + 900 = 300$$

$$-75x = -600$$

$X = 8$ So both tanks have the same amount of water at 8 hours. If we substitute 8 into the equations, we get $y = 500$ for both equations so at 8 hours both tanks have 500 gallons of water.