

Victor and Sharon's Road Trip

6th Grade

Overview: In this lesson, students determine whether two vehicles arrive at a given destination at the same time or whether one arrives before the other, given their rates of speed. An extension asks them to determine their relative arrival times at a generalized destination.

Mathematics: To solve the first task successfully, students can approach it by either determining how long each vehicle will take to reach the given point, or they can compare the rates directly. The extension can only be solved by a comparison of the two rates of speed. These comparisons can be made using a factor-of-change approach, a unit-rate method or a cross multiplication approach.

Goals:

- Students will solve the problem using a variety of strategies.
- Students will be able to use rates to determine time given distance.
- Students will develop an understanding of rates as a multiplicative comparison of two measures.
- Students will be able to explain the equivalence of 8:10 and 20:25 using a variety of methods.

Sixth Grade Content Standards:

- NS 1.2** Interpret and use ratios in different contexts (e.g. batting averages, miles per hour) to show the relative size of two quantities.
- AF 2.2** Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.
- AF 2.3** Solve problems involving rates, average speed, distance, and time.

Building on Prior Knowledge: Fifth Grade Content Standards

- NS 1.2** Find decimal and percent equivalents for common fractions and explain why they represent the same value.

Building on Prior Knowledge: Fifth Grade Content Standards (Cont'd.)

To solve this task successfully, students need to understand that a rate is a comparison of the measures of two different quantities, measured by two different units, and both quantities must be taken into consideration in a coordinated fashion. The context of the problem facilitates various intuitive approaches to ratio and rate, which should be encouraged. Some students may determine unit rates (how far each vehicle would travel in one unit of time, or how long it will take each vehicle to travel one unit of distance) and will find that both rates are equivalent. Alternatively, by using a factor-of-change approach (which is equivalent to finding common denominators), students could find that 8 miles per 10 minutes and 20 miles each 25 minutes are both equivalent to 4 miles every 5 minutes, and both will take 10 minutes to travel 8 miles. Ratios (and rates) also provide students with another context to apply the skills they used when working with fractions.

Materials: Victor and Sharon's Road Trip Task (attached), calculators

From NAEP, 8th grade, NCES – National Center for Education Statistics

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P	<u>HOW DO YOU SET UP THE LESSON?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.	<u>HOW DO YOU SET UP THE LESSON?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.
S E T U P	SETTING THE CONTEXT FOR THE TASK Ask students to follow along as you read the problem. Victor's van travels at a rate of 8 miles every 10 minutes. Sharon's sedan travels at a rate of 20 miles every 25 minutes.	SETTING THE CONTEXT FOR THE TASK It is important that students have access to solving the problem from the beginning.
S E T U P	If both cars start at the same time, will Sharon's sedan reach point A, 8 miles away, before, at the same time, or after Victor's van? Explain your reasoning. If both cars start at the same time, will Sharon's sedan reach point B (at a distance further down the road) before, at the same time, or after Victor's Van? Explain your reasoning.	• Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. • Check on students' understanding of the task by asking several students what they know and what they are trying to find when solving the problem. • Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P S E T U P	SETTING UP THE EXPECTATIONS FOR DOING THE TASK Remind students that they will be expected to: • justify their solutions in the context of the problem. • explain their thinking and reasoning to others. • make sense of other students' explanations. • ask questions of the teacher or other students when they do not understand. • use correct mathematical vocabulary, language, and symbols. Tell students that their groups will be expected to share their solutions with the whole group using the board, the overhead projector, etc.	SETTING UP THE EXPECTATIONS FOR DOING THE TASK Setting up and reinforcing these expectations on a continual basis will result in them becoming a norm for the mathematics classroom. Eventually, students will incorporate these expectations into their habits of practice for the mathematics classroom.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> Circulate among the groups as students work privately on the problem. Allow students time to individually make sense of the problem.	<u>INDEPENDENT PROBLEM-SOLVING TIME</u> It is important that students be given private think time to understand the problem for themselves and to begin to solve the problem in a way that makes sense to them.
E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> Ask questions such as: • What are you trying to find? • What can you tell me about Victor? What can you tell me about Sharon? • Is it important to know that Victor and Sharon start at the same time? • How will you decide whether Sharon and Victor will arrive at the same time, or whether one will arrive before the other?	<u>FACILITATING SMALL-GROUP EXPLORATION</u> <u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method.
E X P L O R E	<u>Possible Solution Paths</u> Strategies will be discussed as well as the questions that you might ask students and the misconceptions that you might encounter. Representations of these solutions are included at the end of this document.	<u>Possible Solution Paths</u> Questions should be asked based on where the learners are in their understanding of the concept. It is important that student responses are given both in terms of the context of the problem and in correct mathematical language, with particular emphasis on always including the units in their explanations.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
EXPLORE	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using an algorithm or procedure (cross multiplication):</i> Ask questions such as: - Who arrives first, Victor or Sharon? How can you be certain? - How do the two products relate to the problem situation? - How long does it take Sharon to travel 8 miles? How can you be certain that your answer is correct? - How could you find the solution to the problem without using cross multiplication?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using an algorithm or procedure (cross multiplication):</i> Algorithmic solutions are those based on learned procedures; unfortunately students often implement these without understanding why or what they are doing. The methods described in the next sections are more intuitive and should be encouraged at this level. Cross-Multiplication is sometimes taught as a procedure for determining the equivalence of fractions or ratios, and students who have learned it previously may apply it in this case (Note: the equality of the ratios is what we are trying to determine – we do not know it prior to performing the procedure): $\frac{8}{10} = \frac{20}{25}$ $8 \times 25 = 10 \times 20 = 200$, therefore the two ratios are equivalent. Possible Student Responses - Students should state that Victor and Sharon will arrive at the same time since the rates are the same. - Students may be unable to explain the relationship between 200 and the problem situation, though it means that each driver would travel 200 miles in 250 minutes (see discussion at the end) - It will be difficult for students to decide how long it will take Sharon to travel 8 miles using this method. This may encourage them to try other methods. - It is hoped that your questioning will help students to see the limitations of this method and will be encouraged to try a more intuitive approach.
EXPLORE		
EXPLORE		

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u>	<u>FACILITATING SMALL-GROUP EXPLORATION (Cont'd.)</u>
		<i>Possible misconceptions or errors:</i> In terms of our problem, students who cross multiply will probably not understand what the 200 represents, and will not be able to explain how this shows that the two drivers are traveling at the same speed. This process also does not help them to determine how long it will take Sharon to drive 8 miles. A more detailed discussion of cross multiplication is given below.
	<i>Using unit-rate methods (overview):</i> <i>Possible misconceptions or errors:</i> - Students might focus on only one of the two quantities in the ratio instead of considering them both. - Students sometimes attempt to use an additive strategy to compare ratios instead of a strategy that uses multiplication or division. Ask questions such as: - What does it mean to travel 8 miles every 10 minutes and 20 miles every 25 minutes?	<i>Using unit-rate methods (overview):</i> Unit rate methods are built on an understanding of the meaning of “every” in rates, and the multiplicative nature of ratio and rates. <i>Possible misconceptions or errors:</i> - Students might attempt to reach a conclusion by comparing only the times, or the distances of the two drivers. For example a student might say that Sharon takes 25 minutes while Victor takes 10 minutes, so she is slower; or Sharon travels 20 miles while Victor only travels 8 miles, so Sharon is faster. While it is common for students to use additive strategies to compare ratios, there might be less of a tendency to do this with rates because rates relate two different types of units – in this case we are given the number of miles traveled in a given number of minutes. - An example of an additive misconception would be for a student to reason that since Victor takes two more minutes than miles to travel, and Sharon takes 5 more minutes than miles, that Sharon is slower.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using a unit-rate method to compare the times it takes to travel one mile:</i> Ask questions such as: • How did you get 1.25? • What does 1.25 mean? • Who will arrive at Point A first? How do you know? • How could you decide who would arrive at Point B first, if you don't know how far it is?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using a unit-rate method to compare the times it takes to travel one mile:</i> Possible Student Responses • Victor travels 8 miles every 10 minutes. This means he travels 1 mile every $10/8$ minutes, or 1 mile every 1.25 minutes. • Sharon travels 20 miles every 25 minutes means she travels 1 mile every $25/20$, or 1 mile every 1.25 minutes. • Since it takes them both 1.25 minutes to travel 1 mile, they are both traveling at the same rate and would both reach Point A at the same time. • Since the rates are the same, they would also reach Point B at the same time. The distance doesn't matter.
	<u>E X P L O R E</u> <i>Using a unit-rate method to compare the distances traveled in a unit time:</i> Ask questions such as: • How did you get .8? • What does .8 mean? • Who will arrive at Point A first? How do you know? • How could you decide who would arrive at Point B first, if you don't know how far it is?	<i>Using a unit-rate method to compare the distances traveled in a unit time:</i> Possible Student Responses • Since Victor travels 8 miles every 10 minutes he travels $8/10$ miles, or .8 miles, in 1 minute. • Since Sharon travels 20 miles in 25 minutes she travels $20/25$ miles, or .8 miles, in 1minute. • Since they both travel the same distance in one minute, they are traveling at the same rate and would both reach Point A at the same time. • Since the rates are the same they would also reach Point B at the same time. The distance doesn't matter.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using a unit-rate method to determine time of arrival:</i> Ask questions such as: • How did you know that it would take Victor 10 minutes to arrive at Point A? • How did you determine how long it would take for Sharon to arrive at Point A? • How could you decide who would arrive at Point A first? • How could you decide who would arrive at Point B first, since you don't know how far it is?	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using a unit-rate method to determine time of arrival:</i> To determine who arrives at point A first, students do not have to compare rates since they can calculate each arrival time and then compare them. • Students should state that we are told that Victor travels 8 miles in 10 minutes, therefore, he will arrive at Point A in 10 minutes. • Students should state that since Sharon travels 20 miles every 25 minutes, this means she travels 1 mile every 25/20 minutes, or 1.25 minutes. To find how long it will take her to travel 8 miles, you multiply 8 x 1.25, therefore she will arrive in 10 minutes. • Since they both take 10 minutes to travel 8 miles, they will both arrive at Point A at the same time. • Since they both are traveling at the same rate, they will also arrive at Point B at the same time.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
E X P L O R E	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using a factor-of-change method:</i>	<u>FACILITATING SMALL-GROUP EXLORATION (Cont'd.)</u> <i>Using a factor-of-change method:</i> The factor-of-change method is a more spatial, or physical, interpretation of the context. Students may understand that a trip can be broken down into equal sized “chunks”. There may also be situations where students will envision the trips being continued in equal-sized chunks, for instance, if Point C were 100 miles away. They will either divide or multiply both parts of the rate by the same number.
E X P L O R E	Ask questions such as: • How do you know that Victor travels 4 miles in 5 minutes? • How do you know that Sharon travels 4 miles in 5 minutes? • How did you decide which numbers to divide by? • How could you decide who would arrive at Point A first? • How could you decide who would arrive at Point B first, since you don't know how far it is?	• We know that Victor travels 8 miles in 10 minutes. You can break the trip into equal parts – we can divide the trip into two parts so we can divide both the miles and minutes by 2. Therefore he travels 4 miles in 5 minutes. • We know that Sharon travels 20 miles in 25 minutes. We can divide her trip into 5 equal parts. We divide both miles and minutes by 5 and find that she travels 4 miles in 5 minutes. • Students should state that they are trying to break both trips up into smaller chunks. They are looking for common factors that can be used in each case. Both 8 and 10 are divisible by 2, therefore, Victor's trip can be broken into two chunks and each is 4 miles in 5 minutes. Since both 20 and 25 are divisible by 5, Sharon's trip can be broken into five chunks and each is also 4 miles in 5 minutes. • Since they both travel 4 miles in 5 minutes, they travel the same rate and will arrive at Point A at the same time. • Since we found that they both travel the same speed, they will always arrive at the same time.
E X P L O R E		

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> Students may use both the factor-of-change and unit-rate methods. Some students might also use the cross-multiplication method. <u>Possible Solutions to be Shared</u> <i>Using an algorithm or procedure (cross multiplication):</i> If no student tried this method, it is not necessary to discuss it at this time, since it does not reinforce the conceptual meaning of rate. However, if this method was seen, it is important to push for the meaning of the 200, which students might not understand. If you are concerned that this might embarrass a student, you should have this solution prepared ahead of time and say, “When I did this problem with another class, one of the groups produced this solution.” Ask questions such as: • What were we trying to find in this problem? How will cross multiplication help us to find this? • What does the 200 mean? • Does this method help us to find out how long it will take Sharon to travel 8 miles?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</u> <u>What solution paths will be shared, in what order, and why?</u> The order in which solutions are shared can assist the teacher in making certain the goals of the lesson are achieved. Starting with a solution path used by most or many of the students allows students access to the discussion. <u>Possible Solutions to be Shared</u> <i>Using an algorithm or procedure (cross multiplication):</i> Starting with a solution that contains errors or misconceptions allows the misconceptions or errors to be addressed publicly in case other students or groups had the same misconception or error. Possible Student Responses • Students should state that we were trying to find out who would arrive at Point A first. Cross multiplication showed that the two rates were equal, and therefore Sharon and Victor would arrive at the same time. • Students may not understand the meaning of 200, however a student might show that both $8/10$ and $20/25$ are equivalent to $200/250$, therefore both Sharon and Victor would travel 200 <i>miles</i> in 250 <i>minutes</i>. • Students will not be able to find Sharon's travel time using only this method, and this will help to set up the motivation for the other methods.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a unit-rate method to compare the times it takes to travel one mile:</i> Ask questions such as: • Why did you divide 10 by 8, and 25 by 20? • What does 1.25 mean? • How does this help us to compare the rates of Victor and Sharon? • How can we use this method to find out how long it takes Sharon and Victor to travel 8 miles?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a unit-rate method to compare the times it takes to travel one mile:</i> This solution is one that helps students make sense of the rate idea and is a good contrast to the cross multiplication method, which is often performed without understanding. Possible Student Responses • Students should state since it took Victor 10 minutes to travel 8 miles, you would divide 10 by 8 to find out how long it takes to travel one mile. Dividing 25 by 20 will show us how long it will take Sharon to travel one mile. • Students should state that this means that Victor and Sharon each take 1.25 minutes to travel one mile. It is important that they give the units in their discussion. • Students should state that this shows that both Victor and Sharon travel at the same rate since they both take 1.25 minutes to travel 1 mile. • This question serves to push students further, particularly if they had only focused on the equality of the rates. It will be addressed below.

Phase	TEACHER PEDAGODY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a unit-rate method to compare the distances traveled in a unit time:</i> Ask questions such as: • Why did you divide 8 by 10, and 20 by 25? • What does .8 mean? • How does this help us to compare the rates of Victor and Sharon? • How does this method relate to the previous method? • Why can we solve the problem in both ways?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a unit-rate method to compare the distances traveled in a unit time:</i> The final two questions help students see that they can meaningfully invert ratios and rates. It is important for them to also invert the units. You might also want to show that they could not do this with fractions – e.g. $\frac{1}{2}$ and $\frac{2}{1}$ are not equivalent quantities. Possible Student Responses • Students should state since Victor traveled 8 miles in 10 minutes, to find how far he traveled in one minute you divide 8 by 10. Since Sharon traveled 20 miles in 25 minutes, you divide 20 by 25 to find out how far she traveled on one minute. • Students should state that this means that Victor and Sharon each traveled .8 miles in one minute, and thus they traveled at the same rate. It is important that they give <i>both</i> units in their answer. • Students should state that this shows that both Victor and Sharon travel at the same rate since they both traveled .8 miles in 1 minute. • Students should state that, in the previous method, they were trying to find out how long it took to travel one mile, while this time they are trying to find out how many miles they can travel in one minute. • Students should state that they can look at the problem in either way – Victor travels 8 miles in 10 minutes, and it takes him 10 minutes to travel 8 miles. Both statements say the same things since we are keeping track of the units.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a unit-rate method determine the time of arrival:</i> Ask questions such as: • Since Victor and Sharon both took 1.25 minutes to travel 1 mile, how did you determine how long it would take them to travel 8 miles? • How did you know that you should multiply by 8? • Since Victor and Sharon both travel .8 miles in 1 minute, how did you find how long it would take them to travel 8 miles? • How did you decide to multiply by 10?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a unit-rate method to determine the time of arrival:</i> It is possible to answer both parts of the problem without calculating the time of arrival. Determining the time of arrival, however, provides a good opportunity to meaningfully apply multiplication in a problem setting. Possible Student Responses • Students should state that since they travel 1 mile in 1.25 minutes, you multiply 1.25 by 8 to find out how long it takes to travel 8 miles. • Students should state that since <i>each</i> mile takes 1.25 minutes, they are taking 1.25 minutes 8 times when they travel 8 miles. • Students should state that since .8 miles times 10 is 8 miles, it takes 1 x 10 or 10 minutes to travel 8 miles. • Some students may know that .8 times 10 is 8. Others may use repeated addition to find how many .8's are in 8, or may use trial and error to find the number that you multiply by .8 to get 8. This will tell them the number of minutes it will take to travel 8 miles.

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S H A R E D I S C U S S A N D A N A L Y Z E	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a factor-of-change method:</i> Ask questions such as: • How do you know that you can think of Victor's rate as 4 miles in 5 minutes and Sharon's rate as 4 miles in 5 minutes? • Why did you divide Victor's trip into two parts and Sharon's trip into five parts? • How do you know that Victor and Sharon arrive at Point A at the same time? • How did you figure out that both Sharon and Victor would arrive at Point B at the same time? <u>HOMEWORK</u> If Sandy drives at a rate of 12 miles every 20 minutes, how long will it take her to drive 24 miles? 8 miles? 18 miles? How far will she drive in 40 minutes? 10 minutes? 25 minutes?	<u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (Cont'd.)</u> <i>Using a factor-of-change method:</i> It is important to stress the "chunking" implied by this method. Encourage students to make some sort of spatial representation of Sharon's trip to show that they can it into equal segments. This will also help them to see that looking at the rate in smaller chunks is like simplifying a fraction. Possible Student Responses • Students should state that the rate tells you how long it takes Victor and Sharon to travel a given distance. Since they are traveling at a steady rate, you can look at smaller parts of this trip by dividing both the distance and time into the same number of parts. • Students should say that they noticed that if they divided Victor's initial trip into two parts and Sharon's into five parts, that you would see that they had both traveled the same distance in the same time. • Students can either state that since their rates are the same they arrive at the same time, or they might notice that since they travel 4 miles in 5 minutes, they will both travel 8 miles in 10 minutes and thus will arrive at the same time. • Students should state that since Victor and Sharon travel the same distance in the same time, and since they are traveling at a steady rate, that they will arrive at all places at the same time. <u>HOMEWORK</u> The next day, have students share the different methods they used to answer these questions.

More on Cross Multiplication

Why does cross multiplication allow us to determine whether two ratios are equivalent?

To understand why cross multiplication works, let's rename both ratios as ratios with a common denominator. We know that one way to find a common denominator (not necessarily the LCM) is to find the product of the two denominators – in our case 10×25 , which equals 250. To express $8/10$ as an equivalent ratio with a denominator of 250, we must multiply the ratio by $25/25$. This is like scaling up our ratio using a factor of 25. Likewise, to express $20/25$ as an equivalent ratio with a denominator of 250, we must multiply it by $10/10$. We now see that both ratios are equivalent to $200/250$, and thus they are equal.

$$\frac{25}{25} \times \frac{8}{10} = \frac{20}{25} \times \frac{10}{10} \quad ?$$

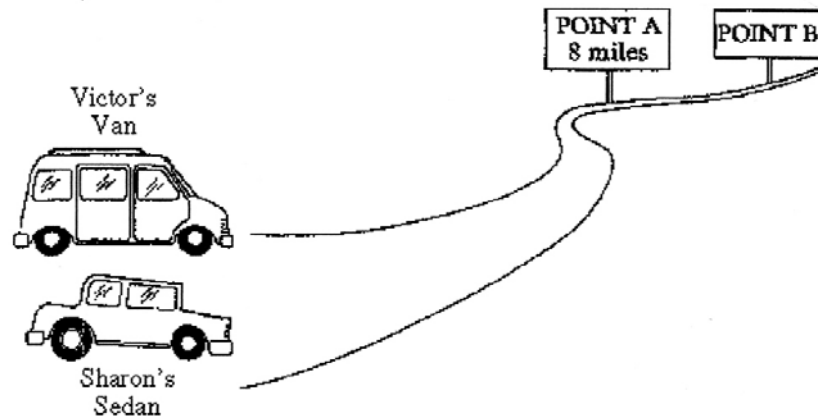
$$\frac{200}{250} = \frac{200}{250}$$

While this method “works”, it may not be clear to the student what the 200 means, since the denominators (250) are not represented. In the context of our problem we have found that both drivers will travel 200 miles in 250 minutes, though this is “hidden” from the student since the denominator of the ratio is not shown. Cross multiplication allows us to directly compare the numerators when we know that we have expressed both ratios using common denominators.

Another problem arises when students learn the cross multiplication process as an isolated procedure. They will often try to use cross-multiplication in situations where it does not apply, such as the multiplication of fractions. For example, when faced with a problem such as

$\frac{2}{3} \times \frac{4}{5}$ some students will apply an erroneous cross-multiplication process.

An additional shortcoming of this method is that while it can also be used to compare two unequal fractions or ratios, it is important that the products are written on the same side of the equation as the numerator in the product. Errors occur when the product is written on the wrong side. Again it would be best to teach students to compare fractions or ratios by other methods that are based on finding equivalent forms (either fractions, ratios, or decimals) that allow the comparisons to be made directly.

VICTOR AND SHARON'S ROAD TRIP

8. Victor's van travels at a rate of 8 miles every 10 minutes. Sharon's sedan travels at a rate of 20 miles every 25 minutes.

If both cars start at the same time, will Sharon's sedan reach point A, 8 miles away, before, at the same time, or after Victor's van?

Explain your reasoning.

If both cars start at the same time, will Sharon's sedan reach point B (at a distance further down the road) before, at the same time, or after Victor's van?

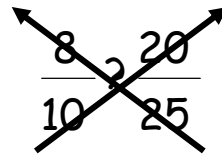
Explain your reasoning.

POSSIBLE SOLUTIONS:

Using an algorithm or procedure (cross multiplication):

Victor travels 8 miles in 10 minutes and Sharon travels 20 miles in 25 minutes. Are their rates equal?

$$\frac{8}{10} = \frac{20}{25} ?$$


$$\begin{array}{ccc} 8 & 20 & \\ \swarrow & \searrow & \\ 10 & 25 & \end{array} ?$$

$$8 \times 25 = 10 \times 20 ?$$

$$200 = 200$$

Therefore, the rates are equal and Sharon and Victor will arrive at the same time.

Using a unit-rate method to compare the times it takes to travel one mile:

How long will it take Sharon and Victor to travel one mile?

Victor takes 10 minutes to travel 8 miles so you divide 10 by 8.

$$\frac{10 \text{ minutes}}{8 \text{ miles}} = 1.25 \text{ minutes for each mile}$$

Sharon takes 25 minutes to travel 20 miles so you divide 25 by 20.

$$\frac{25 \text{ minutes}}{20 \text{ miles}} = 1.25 \text{ minutes for each mile}$$

They both travel at the same rate so they will arrive at the same time.

Using a unit-rate method to compare the distances traveled in a unit time:

How many miles do they travel in 1 minute?

Victor travels 8 miles in 10 minutes, so you divide 8 by 10.

$$\frac{8\text{miles}}{10\text{min}\text{utes}} = 0.8 \text{ miles per minute}$$

Sharon travels 20 miles in 25 minutes, so you divide 20 by 25.

$$\frac{20\text{miles}}{25\text{min}\text{utes}} = 0.8 \text{ miles per minute}$$

They both travel at the same speed so they arrive at the same time.

Using a unit-rate method to determine time of arrival:

How long does it take them each to travel 8 miles?

We were told that Victor travels 8 miles in 10 minutes, so it will take him 10 minutes.

Sharon takes 1.25 minutes to travel 1 mile, so it will take her 1.25×8 minutes to travel 8 miles.

$$1.25 \text{ minutes} \times 8 = 10 \text{ minutes}$$

They both arrive at the same time.

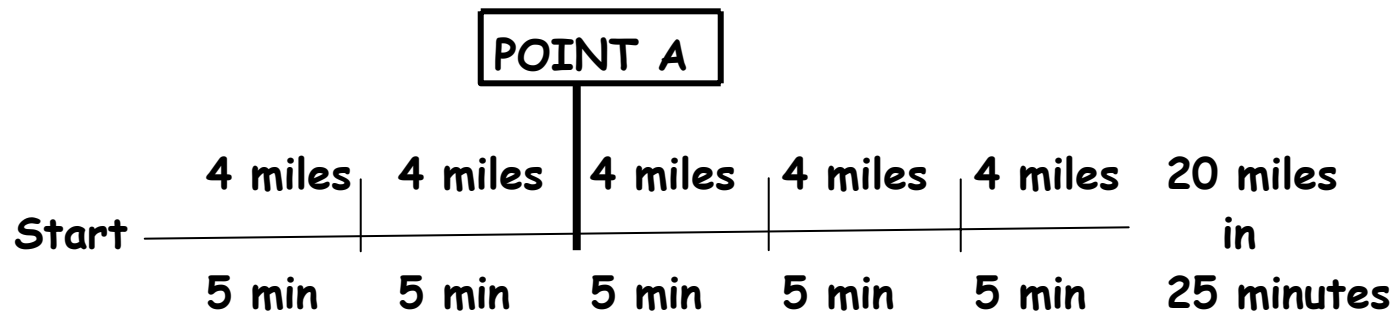
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Another method: Victor and Sharon both travel .8 miles in 1 minute. How long does it take them to travel 8 miles?

Since $.8 \text{ miles} \times 10 = 8 \text{ miles}$, it will take them $1 \text{ minute} \times 10 = 10 \text{ minutes}$.

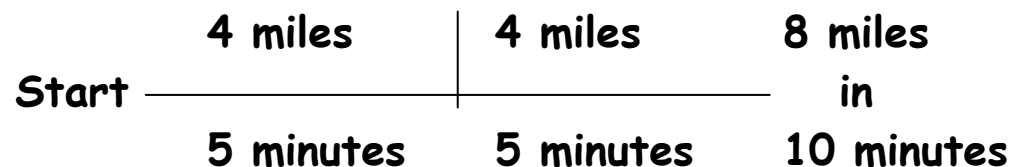
Using a factor-of-change method:

How can I decide how long it takes Sharon to travel 8 miles if she travels 20 miles in 25 minutes? Can I break her trip into "chunks" that will let me figure this out?



I can break the trip into five parts. She travels 4 miles in 5 minutes so it takes her 10 minutes to arrive at Point A, 8 miles.

How can I find out if Victor and Sharon are traveling at the same rate using "chunks"? Victor travels 8 miles in 10 minutes



I can break Victor's trip into two parts, and I see that he also travels 4 miles in 5 minutes, so they both travel at the same rate.