

## Concept Lesson: Measuring Toy Boxes

### Third Grade – Quarter 4

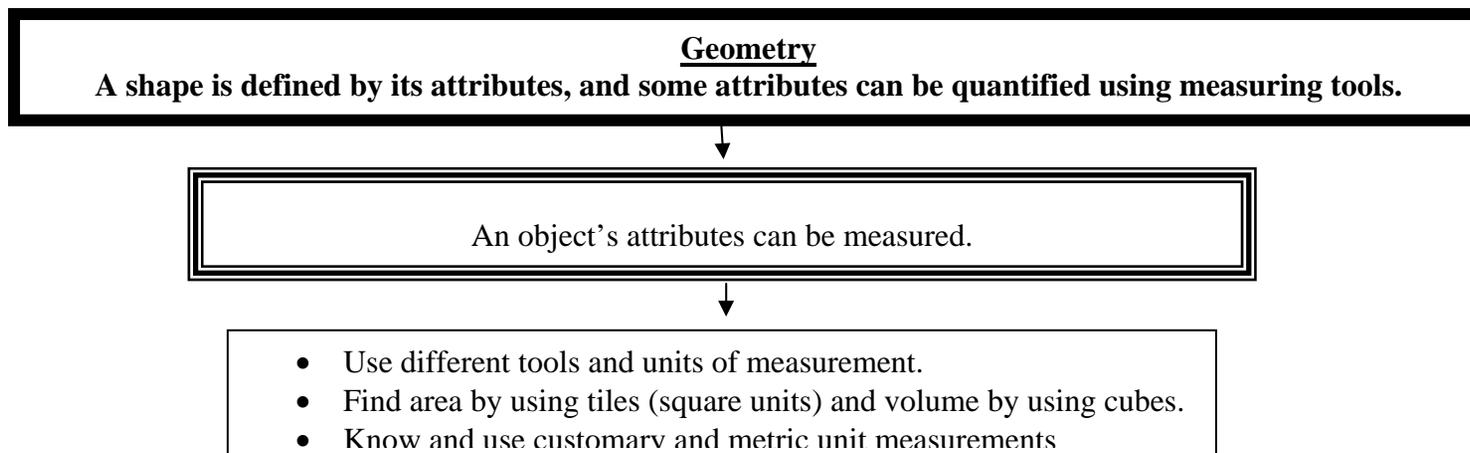
**Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems, which utilize the concepts or skills in question.**

#### **Student Task:**

In this lesson, students will develop strategies for finding the volume of 2 rectangular prisms. They will decide who has the larger toy box as they use multiplicative reasoning and develop an understanding of cubic measurement to determine the number of cubes that would fill a  $4 \times 3 \times 2$  rectangular prism and a  $5 \times 2 \times 3$  rectangular prism.

#### **Materials:**

- Cubes (base-ten units or other cubes; 54 per student or pair of students); task sheet (attached); nets of each toy box (to be cut out and assembled; optional); transparencies or chart paper for selected students to record their solutions; overhead markers or markers; pictures of toy boxes or other boxes used for storage (optional)



#### **Standards Addressed in the Lesson:**

- |               |  |
|---------------|--|
| <b>MG 1.1</b> | <b>Choose the appropriate tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.</b>  |
| <b>MG 1.2</b> | <b>Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.</b>   |
| <b>MR 1.2</b> | <b>Determine when and how to break a problem into simpler parts.</b>   |
| <b>MR 2.2</b> | <b>Apply strategies and results from simpler problems to more complex problems.</b>  |
| <b>MR 2.4</b> | <b>Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.</b> |
| <b>MR 3.2</b> | <b>Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.</b>  |
| <b>MR 3.3</b> | <b>Develop generalizations of the results obtained and apply them in other circumstances.</b>  |

### **Mathematical Concept Goals:**

The mathematical concept goals addressed in this lesson:

- **Develop strategies for finding the volume of rectangular prisms.**
- **Develop an understanding of the concept of volume.**

### **Academic Language**

The concepts represented by these terms should be reinforced/developed through the lesson:

- |                |                     |        |
|----------------|---------------------|--------|
| • Volume       | • Width             | • Base |
| • Layer        | • Height            | • Cube |
| • Dimension(s) | • Cubic Units       |        |
| • Length       | • Rectangular Prism |        |

Encourage students to use multiple representations (drawings, manipulatives, diagrams, words, number(s)) to explain their thinking.

### **Assumption of prior knowledge/experiences:**

- Basic knowledge of concepts of multiplication with single-digit factors.
- Understanding of the characteristics of a rectangular prism.
- Experience filling rectangular prisms with cubes.

### **Organization of Lesson Plan:**

- The left column of the lesson plan describes rationale for particular teacher questions or why particular mathematical ideas are important to address in the lesson.
- The right column of the lesson plan describes suggested teacher actions and possible student responses.

### **Key:**

**Suggested teacher questions are shown in bold print.**

*Possible student responses are shown in italics.*

\*\* Indicates questions that get at the key mathematical ideas in terms of the goals of the lesson.

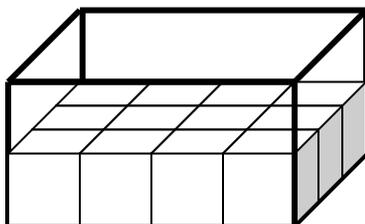
.....  
Essential questions, talk moves, and strategies are highlighted in text boxes like this one in each of the three phases to support and guide teachers, coaches, and administrators as they plan, facilitate, and reflect on the delivery of high-quality concept lessons. These questions, talk moves, and strategies especially support the learning for English Learners, Standard English Learners, Students with Disabilities, and students identified as Gifted and Talented.  
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### **Lesson Phases:**

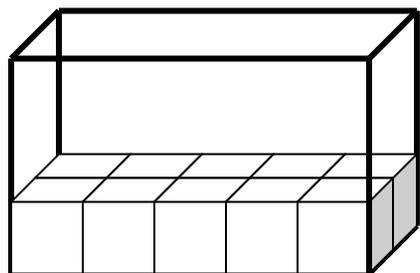
The phase of the lesson is noted on the left side of each page. The structure of this lesson includes the Set-Up; Explore; and Share, Discuss and Analyze Phases.

## Measuring Toy Boxes

Hailee and her brother Jamal can't decide who has a larger toy box, so they use their cubes to measure the base of the toy boxes.



Hailee's toy box is 4 cubes long and 3 cubes wide, and she can put 2 layers of cubes in it.



Jamal's toy box is 5 cubes long and 2 cubes wide, and he can put 3 layers of cubes in it.

Make a prediction of which toy box can hold more cubes.

# Measuring Toy Boxes

## Work Space

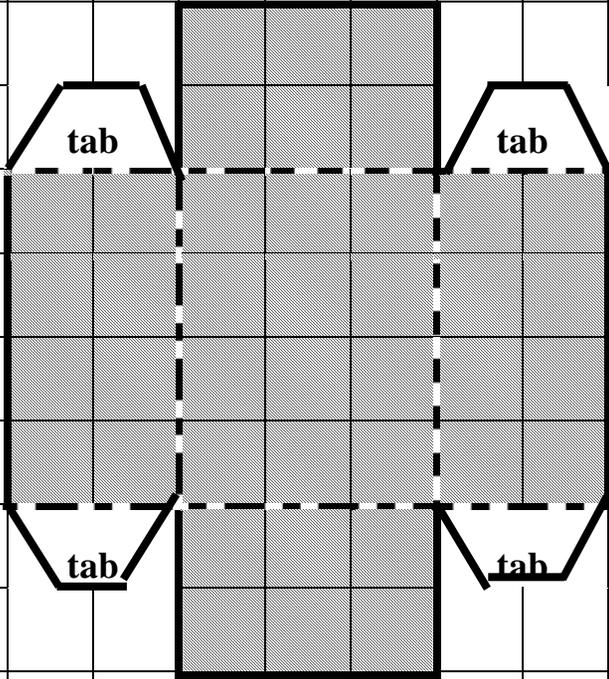
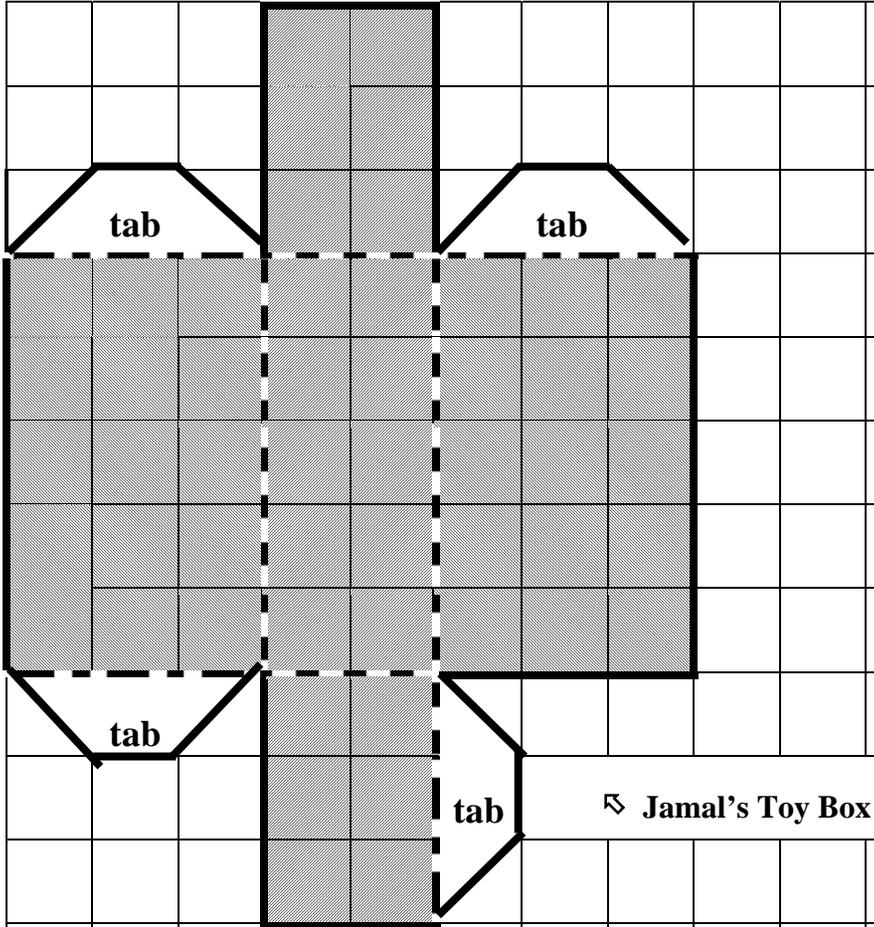
Use pictures, numbers, and words to show how Hailee and Jamal can solve their problem.

# Measuring Toy Boxes

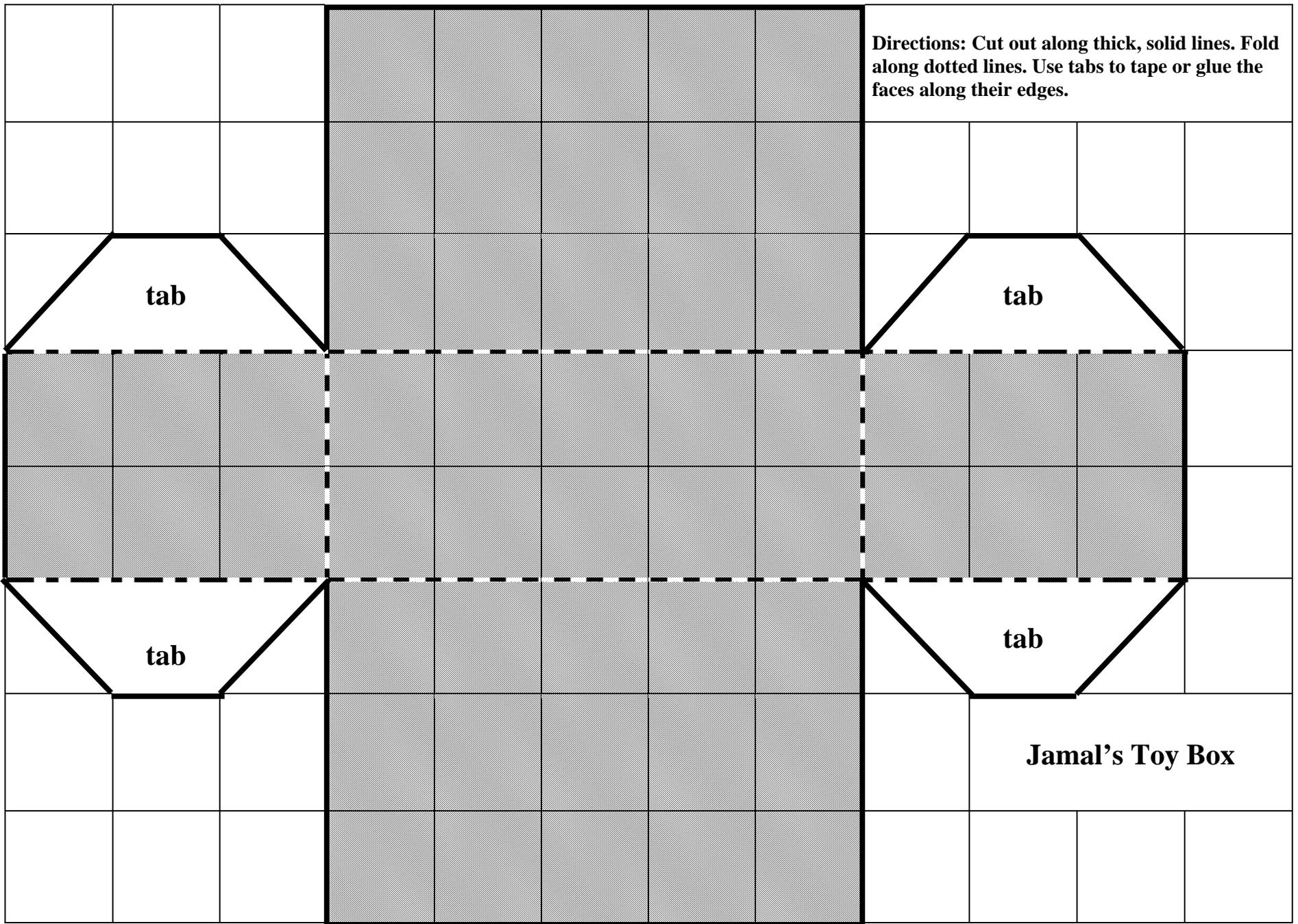
## Lesson Extension

Hailee's cousin, Malia, has a toy box that is 3 cubes long and 3 cubes wide. She can put 3 layers of cubes in it. How does her toy box compare to Hailee's and Jamal's toy boxes? Show your work using pictures, numbers, and words.

**Directions:** Cut out along thick, solid lines. Fold along dotted lines. Use tabs to tape or glue the faces along their edges.

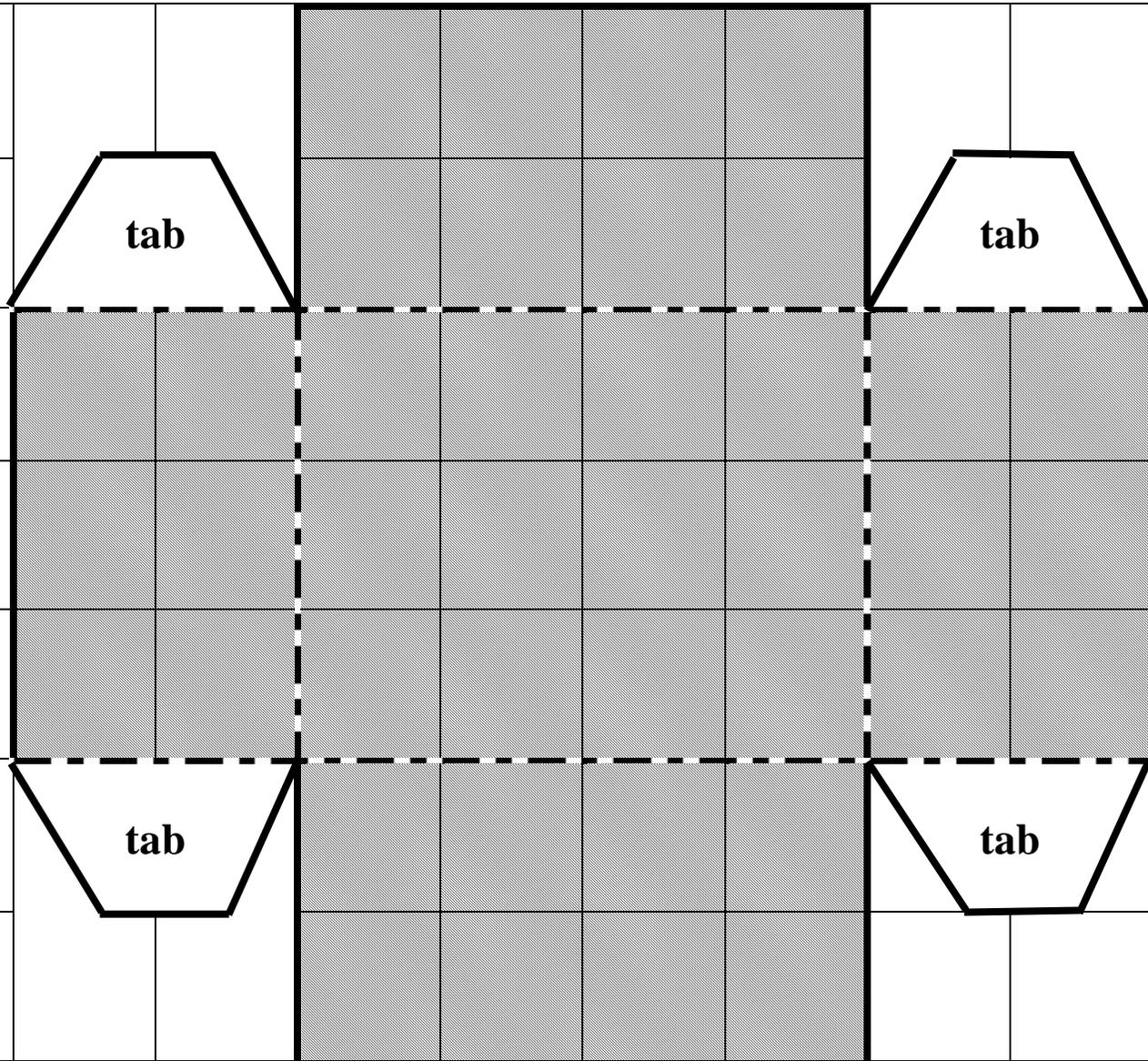


These can be used with centimeter cubes.



**This can be used with 2-centimeter or unifix cubes.**

**Directions: Cut out along thick, solid lines. Fold along dotted lines. Use tabs to tape or glue the faces along their edges.**



**This can be used with 2-centimeter or unifix cubes.**

## THE LESSON AT A GLANCE

### *Set Up (pp. 10-11)*

**Setting up the task:** Solving the task prior to the lesson and providing access to students by strategically pairing students, providing manipulatives, posting key vocabulary terms, and considering how vocabulary will be addressed within the context of the lesson

**Setting the context:** Linking to prior knowledge and establishing a context for the task in order to create real-world connections

**Introducing the task:** Ensuring that students understand what they know and what they are trying to find out



### *Explore (pp. 11-15)*

#### *Independent problem solving time*

#### *Small group exploration:*

- Considering misconceptions that might occur
- Using questioning to guide students who are experiencing difficulty
- Encouraging student-student sharing of and dialogue around solution paths
- Reviewing solution paths, facilitating through questioning, and selecting student work to share



### *Share, Discuss, and Analyze (pp. 16-17)*

#### *Sharing, discussing, and connecting solutions*

#### *Making connections to the dimensions of the rectangular prism*

#### *Considering strategies for determining volume*



### *Summarizing the Mathematical Concepts of the Lesson (p. 19)*

There are a variety of ways that we can find the volume of a rectangular prism. Multiplying the length of a prism by its width tells us the volume of one layer and multiplying that product by its height determines the volume of the entire prism.

## The Lesson

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
<b>S E T U P  S E T U P  S E T U P</b>	<p style="text-align: center;"><b>HOW DO YOU SET UP THE TASK?</b></p> <ul style="list-style-type: none"> <li>• <u>Solving the task prior</u> to the lesson is critical so that:               <ul style="list-style-type: none"> <li>– you become familiar with strategies students may use.</li> <li>– you consider the misconceptions students may have or errors they might make.</li> <li>– you honor the multiple ways students think about problems.</li> <li>– you can provide students access to a variety of solutions and strategies.</li> <li>– you can better understand students’ thinking and prepare for questions they may have.</li> </ul> </li> </ul> <hr style="border-top: 1px dotted black;"/> <ul style="list-style-type: none"> <li>• Planning for how you might help students <u>make connections</u> through talk moves or questions will prepare you to help students develop a deeper understanding of the mathematics in the lesson.</li> </ul> <hr style="border-top: 1px dotted black;"/> <ul style="list-style-type: none"> <li>• It is important that students have <u>access to solving the task</u> from the beginning. The following strategies can be useful in providing such access:               <ul style="list-style-type: none"> <li>– strategically pairing students who complement each other.</li> </ul> </li> </ul> <hr style="border-top: 1px dotted black;"/> <ul style="list-style-type: none"> <li>– providing manipulatives or other concrete materials.</li> <li>– identifying and discussing vocabulary terms that may cause confusion.</li> <li>– posting vocabulary terms on a word wall, including the definition and, when possible, a drawing or diagram.</li> </ul> <hr style="border-top: 1px dotted black;"/> <p style="text-align: center;"><b>SETTING THE CONTEXT FOR THE TASK</b></p> <p><u>Linking to Prior Knowledge</u></p> <p>It is important that the task have points of entry for students.</p> <hr style="border-top: 1px dotted black;"/> <p>By connecting the content of the task to previous knowledge, students will begin to make the connections between what they already know and what we want them to learn.</p> <hr style="border-top: 1px dotted black;"/>	<p style="text-align: center;"><b>HOW DO YOU SET UP THE TASK?</b></p> <ul style="list-style-type: none"> <li>• <u>Solve</u> the task in as many ways as possible <u>prior</u> to the lesson.</li> <li>• Make certain students have <u>access to solving the task</u> from the beginning by:               <ul style="list-style-type: none"> <li>-having students work with a partner or in small groups.</li> <li>-having the problem displayed on an overhead projector or black board so that it can be referred to as the problem is read.</li> <li>-having centimeter or inch cubes on students’ desks.</li> </ul> </li> <li>• Think about how students will understand the concepts used in the task within the context of the lesson.</li> </ul> <hr style="border-top: 1px dotted black;"/> <ul style="list-style-type: none"> <li>• As concepts are explored a word wall can be referenced to generate discussion. The word wall can also be used as a reference if and when confusion occurs.</li> <li>• Think about how you want students to make connections between different strategies.</li> </ul> <hr style="border-top: 1px dotted black;"/> <p style="text-align: center;"><b>SETTING THE CONTEXT FOR THE TASK</b></p> <p><u>Linking to Prior Knowledge</u></p> <ul style="list-style-type: none"> <li>• You might begin by asking students what kinds of boxes they have at home for storage, such as a toy box.</li> <li>• You could also prepare some pictures of toy boxes similar to ones that are in the task and ask:</li> <li>• <b>When buying a toy box or other storage box, what might be important information to have?</b> (<i>How much it will hold; how much space it will take up and the space we have for holding it; etc.</i>)</li> </ul>

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S E T U P  S E T U P  E X P L O R E	<p style="text-align: center;"><b>SETTING THE CONTEXT FOR THE TASK (cont.)</b></p> <ul style="list-style-type: none"> <li>• Having students explain what they are trying to find might reveal any confusions or misconceptions that can be dealt with prior to engaging in the task.</li> <li>• Do not let the discussion veer off into strategies for solving the task, as that will diminish the rigor of the lesson.</li> <li>• Students should be directed to complete the second part of the task once they have made a prediction as to who has the larger toy box, Hailee or Jamal.</li> <li>• The extension problem might be used for early finishers or as a follow-up task to be completed on another day.</li> </ul> <p style="text-align: center;"><b>INDEPENDENT PROBLEM-SOLVING TIME</b></p> <p>It is important that students be given <u>private think time</u> to understand and make sense of the problem for themselves and to begin to solve the problem in a way that makes sense to them.</p> <p><u>Wait time</u> is critical in allowing students time to make sense of the mathematics involved in the problem.</p>	<p style="text-align: center;"><b>SETTING THE CONTEXT FOR THE TASK (cont.)</b></p> <p>Ask a student to read the problem as others follow along:</p> <p>Page 1:</p> <ul style="list-style-type: none"> <li>• <i>Hailee and her brother Jamal can't decide who has a larger toy box, so they use their cubes to measure the base of the toy boxes.</i></li> <li>• <i>Hailee's toy box is 4 cubes long and 3 cubes wide, and she can put 2 layers of cubes in it.</i></li> <li>• <i>Jamal's toy box is 5 cubes long and 2 cubes wide, and he can put 3 layers of cubes in it.</i></li> </ul> <p>Page 2:</p> <ul style="list-style-type: none"> <li>• <i>Use pictures, numbers, and words to show how Hailee and Jamal can solve their problem.</i></li> </ul> <p>• Ask students to think-pair-share what they know and what they are trying to find out.</p> <p>• Ask students to state what they know and what they are trying to find out in this problem. (<i>We know that Hailee's toy box is 4 cubes long and 3 cubes wide and she can put 2 layers of cubes in it. Jamal's toy box is 5 cubes long and 2 cubes wide and he can put 3 layers of cubes in it. We need to predict whose toy box holds more cubes and use pictures, numbers, and words to show how Hailee and Jamal can solve their problem.</i>) Then ask one or two other students to restate what they think they know and what they are trying to find out.</p> <p style="text-align: center;"><b>INDEPENDENT PROBLEM-SOLVING TIME</b></p> <ul style="list-style-type: none"> <li>• Tell students to work on the problem by themselves for a few minutes.</li> <li>• Circulate around the class as students work individually. Clarify any confusions they may have by asking questions but do not tell them how to solve the problem.</li> <li>• After several minutes, tell students they may work with their partners or in their groups.</li> </ul>

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<p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p>	<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p><u>Possible misconceptions or errors:</u></p> <p>It is important to have students <u>explain their thinking</u> before assuming they are making an error or having a misconception. After listening to their thinking, <u>ask questions</u> that will move them toward understanding their misconception or error.</p> <ul style="list-style-type: none"> <li>• Having students demonstrate their thinking using a concrete model often allows them to discover their misconception or error.</li> <li>• Asking students to verify their thinking builds in them the practice of checking their work.</li> </ul> <p><u>Possible misconceptions or errors:</u></p> <ul style="list-style-type: none"> <li>• Encouraging students to share their solutions with each other to the extent that their partner could explain it creates accountability and honors student thinking.</li> <li>• Encouraging students to solve the problem in more than one way builds flexibility of thinking and helps students make connections between models, numbers, and language.</li> </ul>	<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION</b></p> <p><u>If students have difficulty getting started, ask questions such as:</u></p> <ul style="list-style-type: none"> <li>• <b>What do you know? What are you trying to figure out?</b></li> <li>• <b>How can you use the cubes to help you solve the problem?</b></li> <li>• <b>What are some ways that you might try to solve this problem?</b></li> <li>• <b>How can you use a picture to solve the problem?</b></li> <li>• <b>What are some ways that you could use numbers or number sentences to help you solve this problem?</b></li> <li>• <b>What strategy might we use to find the total number of cubes in the bottom layers of each toy box?</b></li> </ul> <p><u>Possible misconceptions or errors:</u></p> <ul style="list-style-type: none"> <li>• <i>Calculation errors when adding or multiplying 12, 2 times and 10, 3 times</i>  <b>Explain how you determined the total number of cubes.</b>  <b>How can you check your work?</b>  <b>What is another way to determine the total number of cubes?</b></li> <li>• <i>Counting only the visible cubes in each toy box or thinking that the toy box with the larger base is larger</i>  <b>What is the problem asking you to do?</b>  <b>What would each toy box look like if it were filled with cubes?</b>  <b>How can you use your cubes to solve the problem?</b></li> <li>• <i>Counting only the cubes that are missing in each toy box</i>  <b>Explain how you determined the total number of cubes.</b>  <b>How do you know your answer is correct?</b>  <b>How do you know that your answer makes sense?</b></li> <li>• <i>Not accounting for the bottom layer: 1 layer of 12 rather than 2 or 2 layers of 10 rather than 3</i>  <b>Explain how you determined the total number of cubes.</b>  <b>How do you know your answer is correct?</b>  <b>How did determining the total number of cubes in the bottom layer assist you?</b></li> </ul>

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<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p style="writing-mode: vertical-rl; transform: rotate(180deg);"><b>E X P L O R E</b></p> <p style="writing-mode: vertical-rl; transform: rotate(180deg);"><b>E X P L O R E</b></p> <p style="writing-mode: vertical-rl; transform: rotate(180deg);"><b>E X P L O R E</b></p>	<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p><u>Possible misconceptions or errors:</u></p> <p>It is important to have students <u>explain their thinking</u> before assuming they are making an error or having a misconception. After listening to their thinking, <u>ask questions</u> that will move them toward understanding their misconception or error.</p> <ul style="list-style-type: none"> <li>• Students should be encouraged to explain their thinking whether their answer is correct or not. Often times, if there is a calculation error, students will correct it as they explain how they arrived at it.</li> <li>• Once students explain their solution path to their partners, they may correct their error and/or clarify a misconception.</li> </ul> <p>Encouraging students to share their solutions with each other to the extent that their partner could explain it creates accountability and honors student thinking.</p> <p>Encouraging students to solve the problem in more than one way builds flexibility of thinking and helps students make connections between models, numbers, and language.</p>	<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION</b></p> <p><u>Possible misconceptions or errors (cont.):</u></p> <ul style="list-style-type: none"> <li>• <i>Counting visible faces in diagram</i> <b>Explain how you determined the total number of cubes. If the volume of the toy box is the total number of cubes, what would you count to find the volume? What is another way to determine the total number of cubes?</b></li> <li>• <i>Adding the 3 numbers next to each of the diagrams</i> <b>Explain how you determined the total number of cubes. What do each of these numbers represent? Why did you add? How can you check your work? How else could you determine the total number of cubes?</b></li> <li>• <i>Adding 2 more additional layers to Hailee’s toy box instead of 1 or 3 more additional layers to Jamal’s toy box instead of 2</i> <b>Explain how you determined the total number of cubes. What does it mean when it says that “she can put 2 layers of cubes” (or “he can put 3 layers of cubes”) in her (or his) toy box? How can you use your cubes to show what each toy box looks like? How else could you determine the total number of cubes?</b></li> <li>• <i>Not understanding concept of layer</i> <b>What does the word “layer” make you think of? Where else have you heard the word “layer”? How can you find the total number of cubes in the bottom layer of the toy box? What does it mean when it says that Hailee can put 2 layers of cubes in her toy box?</b></li> </ul> <p>Additional strategies for addressing misconceptions are embedded within the possible solutions.</p>

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<p><b>E X P L O R E</b></p> <p><b>E X P L O R E</b></p> <p><b>E X P L O R E</b></p>	<p><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p><u>Possible Solution Paths:</u></p> <p>Monitoring students' progress as they are engaging in solving the task will provide you with the opportunity to select solutions for the whole group discussion that highlight the mathematical concepts.</p> <ul style="list-style-type: none"> <li>• <i>Adding 12, 2 times &amp; 10, 3 times <u>or</u> multiplying <math>12 \times 2</math> &amp; <math>10 \times 3</math></i></li> </ul> <p>Students may see that adding the bottom layers 2 times and 3 times (or multiplying by 2 and 3) will yield the total number of cubes for each toy box. If they add, this strategy provides an opportunity to establish the relationship of addition to multiplication. Additionally, this might be a good time to insert the vocabulary of dimensions: width, length, and height to begin helping students make connections to the fact that their answer reflects length x width x height. Do not, though, teach this formula unless the students make note of it in their discussion.</p> <ul style="list-style-type: none"> <li>• <i>Building both toy boxes using cubes and then counting</i></li> </ul> <p>Students might use their cubes to build each of the toy boxes. This will provide an opportunity to discuss the dimensions of each toy box and how what they discovered confirmed or conflicted with their prediction. Also, students should be encouraged to think of other ways they could find the total number of cubes besides counting each cube individually.</p> <ul style="list-style-type: none"> <li>• It is important to consistently ask students to explain their thinking. It not only provides the teacher insight as to how the child may be thinking, but might also assist other students who may be confused.</li> <li>• <i>Adding the number of cubes in each row (4 or 5) 6 times or multiplying <math>4 \times 6</math> or <math>5 \times 6</math></i></li> </ul> <p>Students may decide to add the number of cubes in each row 6 times or multiply 4 (or 5), 6 times. This is again another opportunity to connect repeated addition and multiplication.</p>	<p><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p><u>Possible Solution Paths</u></p> <p>You might ask:</p> <ul style="list-style-type: none"> <li>• <i>Adding 12, 2 times &amp; 10, 3 times <u>or</u> multiplying <math>12 \times 2</math> &amp; <math>10 \times 3</math></i></li> <li>– <math>12 + 12 = 12 \times 2</math> &amp; <math>10 + 10 + 10 = 10 \times 3</math></li> <li>– <b>Explain your thinking.</b></li> <li>– <b>How did you get 12? How did you get 10?</b></li> <li>– <b>What does the 12 (10) represent? What does the 2 (3) represent?</b></li> <li>– <b>Why did you add 12, 2 times and 10, 3 times? What would be another way to write <math>12 + 12</math> and <math>10 + 10 + 10</math>?</b></li> <li>– <b>Why did you multiply <math>12 \times 2</math> and <math>10 \times 3</math>?</b></li> <li>– <b>How do your calculations connect to the pictures of the toy boxes?</b></li> <li>• <i>Building both toy boxes using cubes and then counting</i></li> <li>– <math>24 \text{ cubes} &lt; 30 \text{ cubes}</math>; <i>Jamal's toy box is bigger</i></li> <li>– <b>Explain your thinking.</b></li> <li>– <b>What does each of your models represent?</b></li> <li>– <b>How did you find the total number of cubes that each toy box holds?</b></li> <li>– <b>How else might you find the total number of cubes?</b></li> <li>• <i>Adding the number of cubes in each row (4 or 5) 6 times or multiplying <math>4 \times 6</math> or <math>5 \times 6</math></i></li> <li>– <math>4 + 4 + 4 + 4 + 4 + 4 = 24</math> and <math>5 + 5 + 5 + 5 + 5 + 5 = 30</math></li> <li>– <b>Explain your thinking.</b></li> <li>– <b>What does the 4 (or 5) represent? Why did you add 6 times?</b></li> <li>– <b>What is another way to find the total number of cubes?</b></li> <li>– <b>Where do you see the 4 (or 5) and 6 in the diagrams?</b></li> </ul>

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<p><b>E X P L O R E</b></p> <p><b>E X P L O R E</b></p> <p><b>E X P L O R E</b></p>	<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p><u>Possible Solution Paths (cont.):</u></p> <p>Monitoring students' progress as they are engaging in solving the task will provide you with the opportunity to select solutions for the whole group discussion that highlight the mathematical concepts.</p> <p>Using concrete models helps students test conjectures, deepen conceptual understanding, and make connections to other representations such as symbols and words.</p> <ul style="list-style-type: none"> <li>• <i>Multiplying the length, width, and height of each of the prisms</i></li> </ul> <p>Students may see that multiplying the length by the width and then the height will yield the total number of cubes. This solution can be connected to the one where students may have multiplied the number of cubes in the base by the number of layers (or height). Discussions around equivalence might be facilitated here.</p> <p><u>Advancing Questions</u></p> <p>All students should have opportunities to be advanced in their thinking, as a way to develop more efficient strategies for solving problems, to deepen their understandings, and to make new connections between their understandings.</p> <ul style="list-style-type: none"> <li>• As students begin to discuss their solutions, consider questions you will ask to begin and sustain discussions among them so that they can lead their own developing understanding.</li> </ul>	<p style="text-align: center;"><b>FACILITATING SMALL-GROUP EXPLORATION (cont.)</b></p> <p><u>Possible Solution Paths (cont.)</u></p> <ul style="list-style-type: none"> <li>• <i>Multiplying the length, width, and height of each of the prisms</i> – <math>4 \times 3 \times 2 = 24</math> and <math>5 \times 2 \times 3 = 30</math>; <math>24 &lt; 30</math></li> <li>– <b>Explain your thinking.</b></li> <li>– <b>What do each of your numbers represent?</b></li> <li>– <b>Why did you multiply?</b></li> <li>– <b>Where do you see these numbers in the diagram?</b></li> <li>– <b>What is another way to find the total number of cubes?</b></li> </ul> <p><u>Advancing Questions</u></p> <p><b>What is another way that you might solve this problem?</b></p> <p><b>How can you solve this problem using the cubes?</b></p> <p><b>How can you solve this problem using numbers?</b></p> <p><b>How can you record all of the steps that you took in finding your answer?</b></p> <p><b>What are some other ways that you might use numbers to solve this problem?</b></p> <p><b>How might using a picture have helped you solve this problem?</b></p> <p><b>In what ways are the different strategies that you used to solve the problem the same? How are they different?</b></p> <p>Once most students have finished, get the students' attention and ask them to stop.</p>

Phase	RATIONALE	SUGGESTED TEACHER QUESTIONS/ACTIONS AND POSSIBLE STUDENT RESPONSES
S H A R E  D I S C U S S  A N D  A N A L Y Z E	<p style="text-align: center;"><b>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</b></p> <p><u>How will sharing student solutions develop conceptual understanding?</u></p> <ul style="list-style-type: none"> <li>The purpose of the discussion is to assist the teacher in making certain that students develop a conceptual understanding of volume and different strategies for finding volume. The relationships of the dimensions as well as relationships of repeated addition and multiplication can also be discussed. Questions and discussions should focus on the important mathematics and processes that were identified for the lesson.</li> </ul> <p>.....</p> <p>Connections should be made among solutions to deepen understanding that: 1.) volume is the amount of space a solid figure takes up and is measured in cubic units, 2.) there are a variety of strategies that can be used to find the volume of a rectangular prism, and 3.) multiplication and addition are related. <u>Stop here and mark the importance of sharing student thinking</u> so that students will begin to make connections among each other's work as they build understanding of the concept.</p> <p>.....</p> <p>** Indicates questions that get at the key mathematical ideas in terms of the goals of the lesson.</p> <p><u>Possible Solutions to be Shared and How to Make Connections to Develop Conceptual Understanding:</u></p> <p>.....</p> <ul style="list-style-type: none"> <li>When asking students to share their solutions, the questions you ask should be directed to all students in the class, not just to the student(s) sharing their solution.</li> <li>Students should be expected and encouraged to ask questions of each other and to make connections to their own thinking.</li> <li>Asking students consistently to explain how they know something is true develops in them a habit of explaining their thinking and reasoning. This leads to deeper understanding of mathematics concepts.</li> <li>Asking other students to explain the solutions of their peers builds accountability for learning.</li> </ul> <p>.....</p>	<p style="text-align: center;"><b>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON</b></p> <p><u>How will sharing student solutions develop conceptual understanding?</u></p> <p>The purpose of this first whole group discussion is to provide students opportunities to make connections between various solution strategies.</p> <p><u>Possible Solutions to be Shared and How to Make Connections to Develop Conceptual Understanding:</u></p> <p>Consider how you will make connections between the selected strategies for solving the problem.</p> <ul style="list-style-type: none"> <li><i>Adding 12, 2 times &amp; 10, 3 times <u>or</u> multiplying <math>12 \times 2</math> &amp; <math>10 \times 3</math></i> <ul style="list-style-type: none"> <li><math>12 + 12 = 12 \times 2</math> &amp; <math>10 + 10 + 10 = 10 \times 3</math></li> <li><b>**What do each of the numbers represent?</b></li> <li><b>What connections can you make between your solution and someone else's?</b></li> <li><b>Talk with a neighbor about how this student solved the problem. What strategy was used?</b></li> <li><b>**How do we know that <math>12 + 12</math> (or <math>10 + 10 + 10</math>) is the same as <math>12 \times 2</math> (or <math>10 \times 3</math>)?</b></li> </ul> </li> <li><i>Building both toy boxes using cubes and then counting</i> <ul style="list-style-type: none"> <li><math>24 \text{ cubes} &lt; 30 \text{ cubes}</math>; Jamal's toy box is bigger</li> <li><b>Explain your thinking.</b></li> <li><b>**What connections can you make between how you found the total number of cubes and how _____ found them?</b></li> <li><b>How did you find the total number of cubes that each toy box holds?</b></li> <li><b>**How else might you find the total number of cubes or volume?</b></li> </ul> </li> </ul>

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<p style="text-align: center;"><b>S H A R E</b></p> <p style="text-align: center;"><b>D I S C U S</b></p> <p style="text-align: center;"><b>A N D</b></p> <p style="text-align: center;"><b>A N A L Y Z E</b></p>	<p style="text-align: center;"><b>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (cont.)</b></p> <p><u>Possible Solutions to be Shared and How to Make Connections to Develop Conceptual Understanding:</u></p> <ul style="list-style-type: none"> <li>• <i>Multiplication and Repeated Addition</i></li> </ul> <p>Making connections between multiplication and repeated addition will help students see the usefulness of the former in executing the latter.</p> <ul style="list-style-type: none"> <li>• <i>Adding the number of cubes in each row (4 or 5) 6 times or multiplying <math>4 \times 6</math> or <math>5 \times 6</math></i></li> </ul> <p>Making connections between adding the number of cubes (4 or 5) in each row 6 times and adding the number of cubes in each layer (12 or 10) 2 or 3 times may provide an opportunity to discuss the language of dimensions of a rectangular prism and their measures (length = 4 or 5; width = 3 or 2; height = 2 or 3). Ask questions to help students make those connections.</p> <ul style="list-style-type: none"> <li>• <i>Multiplying the length, width, and height of each of the prisms</i></li> </ul> <p>Some students might see that multiplying all 3 dimensions will yield the total number of cubes or volume. This might be an opportunity to make connections to other solutions that used multiplication. In addition, questions should be asked to elicit that we can measure volume with cubic units.</p> <p>Ask students to think of</p> <ol style="list-style-type: none"> <li>1. what connections they can make to their own solutions and</li> <li>2. the questions they might ask to better understand how the solution shows who has the larger toy box (or the one that holds more cubes.)</li> </ol>	<p style="text-align: center;"><b>FACILITATING THE SHARE, DISCUSS, AND ANALYZE PHASE OF THE LESSON (cont.)</b></p> <p><u>Possible Solutions to be Shared and How to Make Connections to Develop Conceptual Understanding:</u></p> <ul style="list-style-type: none"> <li>• <i>Adding the number of cubes in each row (4 or 5) 6 times or multiplying <math>4 \times 6</math> or <math>5 \times 6</math></i></li> </ul> <p>– <math>4 + 4 + 4 + 4 + 4 + 4 = 24</math> and <math>5 + 5 + 5 + 5 + 5 + 5 = 30</math></p> <p>– <b>What do your numbers represent?</b></p> <p>– <b>What connections can you make between your solution and the one where _____ added 12, 2 times or 10, 3 times (or multiplied 12 by 2 or 10 by 3)?</b></p> <p>– <b>How else could you have determined the total number of cubes or volume?</b></p> <p>– <b>What connections do you see between your strategy and someone else's?</b></p> <ul style="list-style-type: none"> <li>• <i>Multiplying the length, width, and height of each of the prisms</i></li> </ul> <p>– <math>4 \times 3 \times 2 = 24</math> and <math>5 \times 2 \times 3 = 30</math>; <math>24 &lt; 30</math></p> <p>– <b>Explain your thinking.</b></p> <p>– <b>What do each of your numbers represent?</b></p> <p>– <b>Why did you multiply?</b></p> <p>– <b>Where do you see these numbers in the diagrams?</b></p> <p>– <b>What is another way to find the total number of cubes or volume?</b></p> <p>– <b>How is <math>4 \times 3 \times 2</math> the same as <math>12 \times 2</math> (or <math>5 \times 2 \times 3</math> the same as <math>10 \times 3</math>)?</b></p> <p>– <b>What connections do you see between your strategy and someone else's?</b></p>

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<b>S H A R E</b>  <b>D I S C U S S</b>  <b>A N D</b>  <b>A N A L Y Z E</b>	<p style="text-align: center;"><b>SUMMARIZING THE MATHEMATICS OF THE LESSON</b></p> <p>Making connections between the various solution paths will build an understanding of volume as well as equip students with strategies that they can use in future problems. Ask students to consider what information was used in each of the solutions in an effort to make connections to the dimensions of the rectangular prisms and how multiplying them or repeatedly adding the number of cubes in a layer or row aided in finding the total number of cubes.</p> <p>It is important for students to summarize the learning and discuss what new ideas have been gained from their discussion. This builds in accountability to the learning as well as enables the teacher to assess what the students have learned. It also establishes why these concepts are important and helps students make connections to their usefulness in their everyday lives.</p>	<p style="text-align: center;"><b>SUMMARIZING THE MATHEMATICS OF THE LESSON</b></p> <p>You might ask:</p> <ul style="list-style-type: none"> <li>• <b>Based on our discussion today, who has a larger toy box, Hailee or Jamal? How do you know?</b></li> <li>• <b>Based on our discussion today, what new understandings do we have around volume?</b></li> <li>• <b>What new understandings do we have about rectangular prisms?</b></li> <li>• <b>What have you learned that you might be able to use in other problem solving situations?</b></li> <li>• <b>Why is the skill of finding the volume of a box or rectangular prism important?</b></li> <li>• <b>When might we use this in our lives outside of school?</b></li> <li>• <b>How do you think the adults in your life use this skill, finding volume?</b></li> </ul> <p><b><u>Lesson Extension</u></b></p> <p>The lesson extension can be used for early finishers or as another problem to have a second Explore and a Share, Discuss, and Analyze.</p> <p><i>Hailee’s cousin, Malia, has a toy box that is 3 cubes long and 3 cubes wide. She can put 3 layers of cubes in it. How does her toy box compare to Hailee’s and Jamal’s toy boxes?</i></p>

