This past summer you were hired to work at Custom T-Shirts. When a customer places an order for a special design, Custom T-Shirts charges a one-time fee of $15 to set up the design plus $8 for each t-shirt printed.

1. Your first responsibility at Custom T-Shirts is to make a chart that shows how much a customer would be charged for various numbers of shirts. Make a chart that includes the cost of up to 100 shirts.
3. If you have not done so already, write a variable expression that could be used to determine how much to charge a customer for any number of shirts.
4. If a customer wanted to know how many shirts they could order for $100, what would you say? Explain how you figured this out.

**Homework:**
- Fancy T-Shirts charges a one-time fee of $18 to set up the design plus $8 for each t-shirt printed. How will their prices compare to the Custom T-Shirt prices? Explain.
LESSON OVERVIEW

Overview: In this lesson, students first determine the price to buy various numbers of shirts, given the pricing scheme in which there is a one-time fee and a cost per shirt printed. They organize this information to notice patterns and relationships. They then use the information gained to write an algebraic expression and evaluate the expression for larger numbers of shirts. Finally, they work backwards from the total price of an unknown quantity of shirts to solve for the number of shirts. As an extension, students can solve the multi-step linear equation to find the number of shirts that can be purchased for a given dollar amount.

CA Standards Addressed:
Algebra 5.0 - Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Mathematical Goals of the Lesson:
• Develop a deeper understanding of variables and their use in representing mathematical situations.
• Develop an understanding and interpret the meaning of algebraic expressions in one variable that represent mathematical situations.
• Evaluate algebraic expressions in one variable and interpret the results in terms of a mathematical situation.
• Begin to develop the ability to solve linear equations in one variable.
• Reason mathematically, build arguments to support or refute hypotheses, use and make connections among a variety of mathematical representations.

Academic Language Goals of the Lesson:
• Develop academic vocabulary to be used in the descriptions.
• Describe algebraic patterns orally or in writing.
• Explain the process used in solving the task, orally or in writing.

Assumption of Prior Knowledge:
• Recognize that variables represent an unknown quantity.

Academic Language:
• Variable, constant, coefficient
• Variable expression and equation
• Representation
• Rate of change

Materials:
• Task, logo t-shirt
• Calculators
• Chart paper or transparencies, markers
### SET-UP PHASE: Setting Up the Mathematical Task

#### INTRODUCING THE TASK

- Show a t-shirt with a design and ask students what they think a process would be for a company to make t-shirts that have logos or designs on them. Listen for such things as buying the shirts, designing the logo or pictures, and printing the logo or picture on the shirt.
- Ask a student to read the task out loud as others follow along.
- Ask several students to explain what they think they are being asked to do.
- Clarify any confusions students may have, but do not suggest a method for solving the problems.

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**Possible student responses are shown in italics**

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**To assist ELLs’ participation in the class discussion**:  
- **Allow time for students to first talk in small groups (pairs), and then have the groups report to the whole class.**  
- **Reinforce appropriate language as students communicate their ideas (e.g., revoice a student’s contribution in complete, grammatically correct language). Ask students if you have captured what they said.**
<table>
<thead>
<tr>
<th>Phase</th>
<th>EXPLORE PHASE: Supporting Students’ Exploration of the Task</th>
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<tbody>
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<td>STRUCTURE</td>
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</table>

**PRIVATE THINK TIME**
- Ask students to work individually for 5-10 minutes so that they can make sense of the problem for themselves.
- Circulate around the classroom and clarify confusions. Be careful to NOT give away too much information or suggest a way to solve the problem.

**SMALL-GROUP WORK**
- After 5-10 minutes, ask students to work with their partner or in their small groups.
- As students are working, circulate around the room.
  - Be persistent in asking questions related to the mathematical ideas, problem solving strategies, and connections between representations.
  - Be persistent in asking students to explain their thinking and reasoning.
  - Be persistent in asking students to explain, in their own words, what other students have said.
  - Be persistent in asking students to use appropriate mathematical language.

What do I do if students have difficulty getting started?
- Ask: “What would it cost to buy 1 shirt?” “What would it cost to buy a second shirt with the same design?”
- Ask: “If you placed an order and Custom T-Shirts created your design but didn’t print any shirts, how much would they charge you?” “Why?”

What do I do if students finish early?
Ask students to come up with a different price structure that includes a set-up fee and price per shirt but that is always cheaper than the original plan. Have them come up with another price structure that is always more expensive than the original plan.

**MONITORING STUDENTS’ RESPONSES**
- As you circulate, attend to students’ mathematical thinking and to the strategies and representations used, in order to identify those responses that will be shared during the Share, Discuss, and Analyze Phase. For this task, you will need to:
  - Identify a price chart to be used to begin the Share phase. Look for a chart that starts at zero or one shirts, uses a consistent increment, and has correct calculations. Give the group an overhead transparency or chart paper on which to copy their chart. You should have a chart prepared as a backup in case you don’t see such a chart, or if students are working slowly and copying a chart would not be a good use of class time.
  - Identify students who determined price by using a variable expression equivalent to $8n + 15$. Look for different, equivalent, forms of the expression such as $15 + 8n$, $n \times 8 + 15$, etc.
  - Identify students that use verbal strategies to determine total price.
  - Identify strategies students used to correctly determine the number of shirts that can be purchased for $100.
  - Be on the lookout for students who are able to write, solve and explain an equation to find the number of shirts when
given the total price.
## Phase: Explore

### Task Question:
1. Your first responsibility at Custom T-Shirts is to make a chart that shows how much a customer would have to pay for various numbers of shirts. Make a chart that includes the cost of up to 100 shirts.

<table>
<thead>
<tr>
<th>Possible Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look for indicators of students’ understanding:</td>
</tr>
<tr>
<td>• of the base fee being a one-time charge.</td>
</tr>
<tr>
<td>• of the price per shirt.</td>
</tr>
<tr>
<td>• of a systematic organized way of charting information.</td>
</tr>
</tbody>
</table>

### Possible Questions

<table>
<thead>
<tr>
<th>No. of Shirts</th>
<th>Price $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>20</td>
<td>175</td>
</tr>
<tr>
<td>30</td>
<td>255</td>
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<tr>
<td>40</td>
<td>335</td>
</tr>
<tr>
<td>50</td>
<td>415</td>
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<td>60</td>
<td>495</td>
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<tr>
<td>70</td>
<td>575</td>
</tr>
<tr>
<td>80</td>
<td>655</td>
</tr>
<tr>
<td>90</td>
<td>735</td>
</tr>
<tr>
<td>100</td>
<td>815</td>
</tr>
</tbody>
</table>

Ask questions such as:

- How did you find the different prices?
- What do the 0 and 15 mean?
- What patterns do you see in the table?
- So, if you wanted to find the cost of buying more than 100 shirts, what would you do?
- How would you find the cost of buying 1000 shirts?
- Explain in your own words what ___ said (another student).*

### Misconceptions/Errors

- Failing to recognize the 15 as a constant that must be added into the price of the shirts (i.e. they just multiply $8 times the # of shirts).
- Confusing the constant fee (15) and the price per shirt (8) (i.e., using the expression 15N + 8).

### Questions to Address Misconceptions/Errors

- If Custom T-Shirts sets up your design but didn’t print any shirts, how much would it cost you? If they printed 1 shirt, how much would it cost you?
- Suppose Custom T-Shirts had 2 customers – 1 who wants to order 100 shirts and one who wants to order 1000 shirts. Talk with your neighbor to determine what the set-up fee would be for each order, and how much you would pay JUST to print the shirts for each order. Explain your process.*

<table>
<thead>
<tr>
<th>Possible Solutions</th>
<th>Possible Questions</th>
<th>Misconceptions/Errors</th>
<th>Questions to Address Misconceptions/Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look for students who:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>• extended the table.</td>
<td></td>
<td></td>
<td>• Another student says the price of 150 shirts is $1215. How do you think she got that? Can you both be correct?</td>
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<tr>
<td>• gave a verbal or written explanation of their procedure.</td>
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<td></td>
<td></td>
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<tr>
<td>• wrote and evaluated a variable.</td>
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<tr>
<td>• expression equivalent to 8N + 15.</td>
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<td></td>
</tr>
<tr>
<td><strong>Extending a table:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>**No. of Shirts</td>
<td>Price $**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>815</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>895</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>975</td>
<td></td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>1055</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>1135</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1215</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The price for 150 shirts is $1215 because the price increases $80 for every 10 shirts.</td>
<td></td>
<td></td>
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<tr>
<td>Giving a verbal description:</td>
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<tr>
<td>Since Custom T-Shirts charges $8 to print 1 shirt, they will charge 150 times $8 or $1200 to print 150 shirts, and then we have to add on the $15 start-up cost, so 150 shirts will cost you $1215.</td>
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<tr>
<td>Since Custom T-Shirts charges $8 to print 1 shirt, they will charge 750 x $8 or $6000 to print 750 shirts, and then we have to add on the $15 start-up cost, so it will cost $6015 to</td>
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</table>
buy 750 shirts.
<table>
<thead>
<tr>
<th>Phase</th>
<th>Possible Solutions</th>
<th>Possible Questions</th>
<th>Misconceptions/Errors</th>
<th>Questions to Address Misconceptions/Errors</th>
</tr>
</thead>
</table>
| **EXPLORE** | Writing and evaluating a variable expression:  
The cost can be represented by the expression, $15 + 8N$  
The cost for 150 shirts would be $15 + 8(150) = 15 + 1200 = $1215$.  
The cost for 750 shirts would be $15 + 8(750) = 15 + 6000 = $6015$. | You might ask:  
• How did you come up with your expression?  
• What does the N represent? Why did you use a variable to represent that?  
• Explain in your own words what ___ said. (another student)*. | • Writing $8 + 15N$ as the expression  
• Combining the 8 and 15 before multiplying by 150 (i.e., $23 \times 150$).  
• Entering numbers incorrectly into the calculator. | • Look at the problem. What do the 8 and 15 mean? What does N represent? What does $15N$ represent?  
• What would the cost of 10 shirts be? How does this compare to your table? What does the 23 represent?  
• Show me what you put into the calculator. If you were doing this problem without a calculator, how would you do it? |
| **EXPLORE** | Task Question:  
3. If you have not done so already, write a variable expression that could be used to determine how much to charge a customer for any number of shirts. | Same as above                                                                                  | Same as above                                                                          | Same as above                                                                                           |
| **EXPLORE** |                                                                                     | Same as above                                                                                  | Same as above                                                                          | Same as above                                                                                           |

Institute for Learning

Funded by James Irvine Foundation
| Phase | Task Question:  
4. If a customer wanted to know how many shirts they could order for $100, what would you say? Explain how you figured this out. |
<table>
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<tbody>
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<td></td>
<td><strong>Possible Solutions</strong></td>
</tr>
</tbody>
</table>
| **EXPLORE** | Look for indicators of students’ understanding:  
- that they can make use of the information in their table.  
- that they can work backwards (i.e., subtract the one-time fee first) when they are given the total amount of money that they can spend.  
- that they can evaluate the variable expression for various numbers of shirts.  
- that they must “round down” – i.e., “parts of a shirt” should be ignored in this task (i.e., although the calculated answer is 10.625, they can not purchase 0.625 of a shirt). |
| **EXPLORE** | Using the table:  
- Since 10 shirts cost $95, 11 shirts would cost $8 more ($103) which is more than $100. Therefore, you could only purchase 10 shirts.  
- Giving a verbal explanation and working backwards:  
- If I have $100, I’ll have to pay $15 for the set-up fee. So you subtract this from 100, which leaves me $85 for the shirts. If each shirt costs $8, since I know that 10 times $8 = $80 I can buy 10 shirts. The $5 left over is not enough to buy another shirt.  
You might ask:  
- How did you get $103?  
- Why didn’t you try 10.5 shirts?  
- If you purchase 10 shirts with $100, how much money will be left over?  
- Why did you begin by subtracting $15 from the total amount of money?  
- Is there another way you could find how many shirts you could print for $85? |
| **EXPLORE** | • Looking at 100 shirts rather than a cost of $100. |
| **EXPLORE** | • Students might not use the inverse operations (subtraction and division) but instead they might try to add and multiply as they did with the previous problems. |
| **EXPLORE** | • What are we trying to find?  
• What do the columns represent in the table?  
• How much of the $100 will you be able to use to print the t-shirts? What else will you have to pay for (set up)? |
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<th>Questions to Address Misconceptions/Errors</th>
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</thead>
</table>
| **EXPLOR** | **Evaluating the variable expression:** 15 + 8(10) = 95; 15 + 8(11) = 103 Therefore you could only purchase 10 shirts. | You might ask:  
• Tell me what you did.  
• Why didn’t you try 10.5?  
• Can you write an EQUATION that says that the cost of N shirts is $100? | • Students might evaluate the expression for N = 100. | • What are we trying to find? What information have we been given?  
• What does the N represent in the expression? |
Orchestrating the mathematical discussion: a possible Sequence for sharing student work, Key Questions to achieve the goals of the lesson, and possible Student Responses that demonstrate understanding.

Revisiting the Mathematical Goals of the Lesson:
- Develop a deeper understanding of variables and their use in representing mathematical situations.
- Develop an understanding and interpret the meaning of algebraic expressions in one variable that represent mathematical situations.
- Evaluate algebraic expressions in one variable and interpret the results in terms of a mathematical situation.
- Begin to develop the ability to solve linear equations in one variable.
- Reason mathematically, build arguments to support or refute hypotheses, use and make connections among a variety of mathematical representations.

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<tr>
<th>Phase</th>
<th>Sequencing of Student Work</th>
<th>Rationale and Mathematical Ideas</th>
<th>Possible Questions and Student Responses</th>
</tr>
</thead>
</table>
| SHARE | ▪ Begin with the pricing chart that you identified as you monitored students’ explorations. The chart should have correct calculations, should begin with zero or one shirts, and should have consistent increments. | By beginning with a representative pricing chart, you will be able to ground the discussion in a task in which all students engaged. Opening with a general discussion about patterns will provide an entry point for many students. Key mathematical ideas may also surface that you can expand upon later. | ▪ What patterns do you see in the chart?  
  o I see that the numbers are in order in both columns.  
  o I see that all of the prices end in 5.  
  o I see that the number of shirts goes up by ten each time.  
  o I see that the price goes up by $80 each time. It’s always the same.  
  o In my chart the price goes up by $40 and the number of shirts goes up by 5. |
| DISCUSS |  | The RATE OF CHANGE is $8 per shirt, $80 per 10 shirts, $40 per 5 shirts.  
This lesson can lay the foundation for the ideas of slope and y-intercept, which will be developed in future lessons. | ▪ How can we find the $8 in our charts?  
  o I see the $8 in my chart because 10 shirts will cost $80, and my prices jump by 80.  
  o I can see the $8 in my chart because I counted by 5 shirts and 5 shirts cost $40. See – my prices go up by $40.  
  o It’s harder to see the $8 in my chart because I didn’t always go up by the same number of shirts each time.  
  o That’s how I made my chart so fast. I saw that the prices were going up by $80 when my number of shirts went up by 10, so then I just filled in the rest of the chart that way. |
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</table>
| SHARE      | • Continue with a pricing chart in which students extended the table.                        | Continuing with another chart will provide students with a connection to the previous discussion. Students should realize that using the method of adding rows to the chart would take a long time to find the cost for 750 shirts. This is a motivation for evaluating the expression for \( n = 750 \). | • How can we use the chart to find the cost for 150 shirts? I saw someone add the amount for 100 shirts and the amount for 50 shirts. Can we find the cost that way? Why or why not?  
  o That would not work because you’d be including the $15 base fee twice – once for 100 shirts and once for 50 shirts.  
• So, how can we use the chart to find the price for 150 shirts?  
  o You would keep adding 10 shirts and $80 to each row (or $5 and $40) until we get to 150 shirts and $1215.  
• How would we use the chart to find the price for 750 shirts?  
  o We could keep adding rows, but it would take a long time. |
<p>| DISCUSS    |                                                                                              |                                                                                               |                                                                                  |
| AND ANALYZE|                                                                                              |                                                                                               |                                                                                  |</p>
<table>
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</tr>
</thead>
</table>
| SHARE       | • Connect to a student who used an expression.                                                | By introducing the variable expressions, you will be able to link directly to the mathematical    | • What would be another way to find the cost of shirts? Did anyone use a different method to find the cost of 150 shirts?  
| DISCUSS     | • Ask a student to share, and explain, his/her variable expression.                          | sentences and will allow the notion of VARIABLE to be strengthened.                                  | o I figured out that you can just start with 15 and then add that to whatever I get when I take the number of shirts times $8. |
| ANALYZE     |                                                                                             |                                                                                                  | • How can we write that in math symbols? Students might write 15+8N or 8N+15.                              |
                                                                 |                                                                                             |                                                                                                  | o I knew it would be 15 + 8N.                                                                            |
                                                                 |                                                                                             |                                                                                                  | o I know that you have a start-up cost of $15 so you add that, and then I know that each shirt costs $8 so you have to multiply that by N. |
                                                                 |                                                                                             |                                                                                                  | • What does N represent? Why do we multiply N by 8 and add 15?                                          |
                                                                 |                                                                                             |                                                                                                  | o That is how many shirts they ordered. We multiply by 8 because EACH SHIRT costs $8; we add $15 because that is added once to each order. |
                                                                 |                                                                                             |                                                                                                  | • Why did you use a letter to represent the number of shirts?                                           |
                                                                 |                                                                                             |                                                                                                  | o Where they wrote 20 and 50 I wrote N, because I wanted to be able to talk about any number of shirts. We use a letter to stand for any number. We don’t have to know a specific number of shirts before we write the expression. |
                                                                 |                                                                                             |                                                                                                  | • What do we call these letters?                                                                     |
                                                                 |                                                                                             |                                                                                                  | o We call them VARIABLES.                                                                               |
                                                                 |                                                                                             |                                                                                                  | • I heard someone say that 15N + 8 [write on board] described the cost for Custom Shirts. Do you agree or disagree? Why? |
                                                                 |                                                                                             |                                                                                                  | o I disagree, because that says that each shirt costs $15 since we multiply 15 and N, and there is a one-time charge of $8 since we’re adding that on. |
### Phase

<table>
<thead>
<tr>
<th>SHARE</th>
<th>DISCUSS AND ANALYZE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequecing of Student Work</td>
<td>Rationale and Mathematical Ideas</td>
</tr>
</tbody>
</table>

- **SHARE**
  - Have students evaluate the algebraic expression to determine the cost of orders of various numbers of shirts.

- **DISCUSS AND ANALYZE**
  - This will provide more practice in evaluating expressions. Order of operations will be linked to the context of the task.
  - Use the terms **CONSTANT** and **COEFFICIENT** in relation to the context and the expression.
  - **Note**: the COEFFICIENT is the numeric factor of the term 8N. The CONSTANT is the term with no variables.

- **Possible Questions and Student Responses**
  - **Let's look at 15 + 8N. We said the N is called a VARIABLE because it represents the number of shirts we want to purchase. What do the 8 and the 15 mean?**
    - The 15 is a one-time set-up fee. The 8 is the cost of one shirt that we have to multiply times the number of shirts.
  - **We call the 15 the CONSTANT and the 8 a COEFFICIENT. The 15 is the fee that gets added on and the 8 is the price of printing each shirt that is multiplied times the number of shirts.**
  - **Why do you think we call 15 a CONSTANT?**
    - Something is constant if it doesn't change. We have to add 15 to our price no matter how many shirts we buy.
  - **Explain how you found the cost of 20 shirts.**
    - I put 20 into my expression where the N is and got 15 + 8 x 20. I multiplied first because of the order of operations, then I added 15.
  - **Explain how you would find the cost of 750 shirts. 1000 shirts.**
    - I would use my expression and put in 750 or 1000 for N, and then do the calculations.
<table>
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</thead>
</table>
| SHARE DISCUSS | - Ask students to find the number of shirts that can be ordered for $100 by working backwards, supported by a verbal explanation. | Students will work backwards, making sense of the steps for “undoing” their calculation. This will later be linked formally to the steps for solving a multi-step equation. | - (Call on students who explained their solution verbally, and working backwards) **How did you find out how many shirts you could order for $100?**  
  - I took $15 from $100 and got $85. I know that 8 x 10 is 80, so I could buy 10 shirts. I’d have $5 left over.  
- **Why did you first subtract 15 from 100 before you divided by 8?**  
  - Because I knew I had to spend $15 for the one-time fee, so I wouldn’t have all of the $100 to use to print the shirts. |
| ANALYZE CLOSURE | - Have students reflect on the mathematics of the lesson; find links to math that they have explored before; think of tasks that might be related to the big ideas of the lesson. | It is important for students to step back and reflect on the ideas that surfaced and to situate their learning within past experiences, and to think forward to ways that they might build on these ideas in future tasks. This helps them to focus on the interconnectedness of mathematical ideas. | - **What would happen to the prices for Custom T-Shirts if you changed the set-up cost? What would happen to the price if you changed the cost per shirt?** |

**HOMEWORK:**  
Fancy T-Shirts charges a one-time fee of $18 to set up the design plus $8 for each t-shirt printed. How will their prices compare to the Custom T-Shirt prices? Explain.

Have a student read the homework assignment aloud while others read along. Provide an opportunity for students to ask each other, or the teacher, for clarification if needed.

**CLOSURE**  
Pose the following question and allow time for students to discuss it in small groups:
- **What would happen to the prices for Custom T-Shirts if you changed the set-up cost? What would happen to the price if you changed the cost per shirt?**