

**Music Mania
Algebra Lesson Plan
Unit 4 (2005 – 2006)**

LESSON OVERVIEW

Overview:

In this lesson, students analyze two plans for downloading music from the Internet – one represented by a piecewise function and one represented by an exponential function. Students must represent each plan using a table and a graph. They must then explain why each plan is a functional relationship and determine the domain and range of each. NOTE: The point of this lesson is not to write the equations for each plan.

Mathematics in the Lesson:

To solve this task successfully, students must be able to describe why each of the plans represents a functional relationship, both by referring to a graph and by referring to a table of ordered pairs. They must also be able to determine the domain and range of each function, one of which is discrete and one of which is continuous, and both of which have only positive values.

Goals of the Lesson:

- To deepen and broaden students' understanding of functions by exploring two different types of functions -- a piece-wise linear function and an exponential function - and by determining that for each function every value of the independent variable is paired with exactly one value for the independent variable.
- To identify the sets of possible values for the independent and dependent variables for two different functions, to recognize these values as the domain and range of the functions (respectively), and to determine that two functions with different independent variables can not be graphed on the same axis.

Algebra Standards

- 16.0** Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
- 17.0** Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
- 18.0** Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

Academic Language:

The following terms should be reinforced/developed through the lesson:

- Function
- Relation
- Domain
- Range
- Independent Variable
- Dependent Variable

Materials:

- Task (attached)
- Calculator

Assumption of prior knowledge/experience:

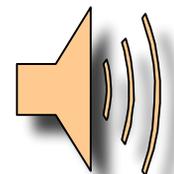
- Use a table to represent a relation
- Use a graph to represent a relation
- Definition of function

Note: Developing an understanding of the mathematical concepts and skills embedded in a standard requires having multiple opportunities over time to engage in solving a range of different types of problems which utilize the concepts or skills in question.

Algebra 1 Unit 4 Task
2005 - 2006



Music Mania



Maria, Ben, and Patrice all received MP3 players as gifts and want to download music from the Internet. They want to find the best possible deal available. Following are two of the plans they found:

1. Super Songs charges 20 cents per minute for the first 100 minutes of downloaded music and then 10 cents per minute for any additional minutes. (You can download fractional parts of a minute.)
2. Terrific Tunes charges by the number of songs downloaded. The first song costs \$10 and each successive song is $\frac{3}{4}$ of the cost of the previous song.

Represent each of the plans using a table and a graph.

1. Do either of the music plans represent a function? Explain why or why not.
2. For each plan, what information would you need to know if you wanted to determine the cost?
3. For each plan, what are the possible values for the cost?
4. Choose the plan that you think is the best deal and justify to Maria, Ben, and Patrice why you chose that plan.

EXTENSION: Represent each plan by writing an equation. Use the equation to determine the domain and range of each.

POSSIBLE SOLUTIONS:

SUPER SONGS

No. Minutes	Cost
0	\$0.00
10	\$2.00
20	\$4.00
30	\$6.00
40	\$8.00
50	\$10.00
60	\$12.00
70	\$14.00
80	\$16.00
90	\$18.00
100	\$20.00
110	\$21.00
120	\$22.00
130	\$23.00
140	\$24.00
150	\$25.00
160	\$26.00
170	\$27.00
180	\$28.00
190	\$29.00
200	\$30.00
210	\$31.00
220	\$32.00
230	\$33.00
240	\$34.00
250	\$35.00
260	\$36.00
270	\$37.00
280	\$38.00
290	\$39.00
300	\$40.00

TERRIFIC TUNES

No. Songs	Cost of Song	Total Cost
0	\$0.00	\$0.00
1	\$10.00	\$10.00
2	\$7.50	\$17.50
3	\$5.63	\$23.13
4	\$4.22	\$27.34
5	\$3.16	\$30.51
6	\$2.37	\$32.88
7	\$1.78	\$34.66
8	\$1.33	\$36.00
9	\$1.00	\$37.00
10	\$0.75	\$37.75
11	\$0.56	\$38.31
12	\$0.42	\$38.73
13	\$0.32	\$39.05
14	\$0.24	\$39.29
15	\$0.18	\$39.47
16	\$0.13	\$39.60
17	\$0.10	\$39.70
18	\$0.08	\$39.77
19	\$0.06	\$39.83
20	\$0.04	\$39.87
21	\$0.03	\$39.90
22	\$0.02	\$39.93
23	\$0.02	\$39.95
24	\$0.01	\$39.96
25	\$0.01	\$39.97
26	\$0.01	\$39.98
27	\$0.01	\$39.98
28	\$0.00	\$39.99
29	\$0.00	\$39.99
30	\$0.00	\$39.99

1. **Do either of the plans represent a function? Why or why not?** Both plans are functional relationships.
 - In the Super Songs plan, the cost depends on the number of minutes that music is downloaded. For any minute value, there is only one cost. The same number of minutes cannot be associated with 2 different costs.
 - In the Terrific Tunes plan, the cost depends on the number of songs downloaded. For any particular number of songs, there is only one cost. The same number of songs cannot be associated with 2 different costs.

2. **For each plan, what information would you need to know if you wanted to determine the cost?** For the Super Songs plan, you need to know the number of minutes that music is downloaded to determine the cost. The domain, the set of possible values for the independent variable, is the set of rational numbers beginning with 0 minutes since fractional parts of a minute are possible. For the Terrific Tunes plan, you need to know the number of songs downloaded to determine the cost. The domain is the set of integers beginning with 0 songs since you cannot download part of a song.

3. **For each plan, what possible values can the cost be?**

The range of values (the set of possible values for the dependent variable) for the cost under the Super Songs plan is from \$0.00 to any amount of dollars and cents since fractional parts of a minute are possible. The range of values for the cost under the Terrific Tunes plan is from \$0.00 to a maximum of \$40 (due to rounding). Since each song is $\frac{3}{4}$ of the cost of the previous song, eventually, the cost of songs becomes \$.00 since money must be rounded to the nearest cent. As a result, the total cost never goes beyond \$40.

4. **Choose the plan that you think is the best deal and justify to Maria, Ben, and Patrice why you chose that plan.**

In order to compare the two plans, you must first establish a basis for comparison by either converting the number of minutes to number of songs or vice versa. For example, if you estimate a song to be 5 minutes long and it is downloaded in real time, then in 300 minutes you could download 60 songs. For \$40, you could download around 60 songs (300 minutes) with Super Songs and 30 songs with Terrific Tunes. So up until 300 minutes, or approximately 60 songs, Super Songs is a better deal. After 300 minutes, or 60 songs, Terrific Tunes would be cheaper because the price does not increase beyond \$40. However, downloading 301 minutes of music with Super Songs would cost \$40.10. **The point of this question is to get student to realize that the two plans have different domains and cannot be directly compared.**

THE LESSON

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S E T U P S E T U P S E T U P	<p><u>HOW DO YOU SET UP THE TASK?</u> Prior to teaching the task, solve it yourself in as many ways as possible. Possible solutions to the task are included throughout the lesson plan.</p> <p>SETTING THE CONTEXT FOR THE TASK You might begin by asking students what they know about downloading music from the Internet. They may be able to describe several plans they know about. You may need to publicly state that we are discussing LEGAL plans for downloading music.</p> <p>Ask students to follow along as you read the problem: Maria, Ben, and Patrice all received MP3 players as gifts and want to download music from the Internet. They want to find the best possible deal available. Following are two of the plans they found:</p> <ol style="list-style-type: none"> 1. Super Songs charges 20 cents per minute for the first 100 minutes of downloaded music and then 10 cents per minute for any additional minutes. (You can download fractional parts of a minute.) 2. Terrific Tunes charges by the number of songs downloaded. The first song costs \$10 and each successive song is $\frac{3}{4}$ of the cost of the previous song. <p>Represent each of the plans using a table and/or graph.</p> <ol style="list-style-type: none"> 1. Do either of the plans represent a function? Explain why or why not. 2. For each plan, what information would you need to know if you wanted to determine the cost? 3. For each plan, what possible values can the cost be? 4. Choose the plan that you think is the best deal and justify to Maria, Ben, and Patrice why you chose that plan. 	<p><u>HOW DO YOU SET UP THE TASK?</u> It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions the teacher may ask students about the problem will be given.</p> <p>SETTING THE CONTEXT FOR THE TASK It is important that students have access to solving the problem from the beginning.</p> <ul style="list-style-type: none"> • Beginning the lesson by relating the task to students' own experience is a nice segue into a lesson • Have the problem displayed on an overhead projector or chart paper so that it can be referred to as you read the problem. • Make certain that students understand the vocabulary used in the problem (i.e. MP3 player, download, successive, function, independent and dependent variables.) • Check on students' understanding of the task by asking several students what they know and what they are trying to find when solving the problem. • Be careful not to tell students how to solve the task, or to set up a procedure for solving the task, because your goal is for students to do the problem solving.

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<p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p>	<p style="text-align: center;">INDEPENDENT PROBLEM-SOLVING TIME</p> <p>It is important that students be given private think time to understand and make sense of the problem for themselves and to begin to solve the problem in a way that makes sense to them.</p> <p style="text-align: center;"><u>FACILITATING SMALL-GROUP EXPLORATION</u></p> <p><u>What do I do if students have difficulty getting started?</u> Ask questions such as:</p> <ul style="list-style-type: none"> • What are you trying to find? • What do you know about each plan? • What is a function? 	<p style="text-align: center;">INDEPENDENT PROBLEM-SOLVING TIME</p> <ul style="list-style-type: none"> • Tell students to work on the problem by themselves for a few minutes. • Circulate around the class as students work individually. Clarify any confusions they may have but do not tell them how to solve the problem. <p style="text-align: center;"><u>FACILITATING SMALL-GROUP EXPLORATION</u></p> <p>Tell students they may now work with their partners. As students continue working, circulate around the classroom.</p> <p><u>What do I do if students have difficulty getting started?</u> It is important to ask questions that do not give away the answer or that do not explicitly suggest a solution method.</p>

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<p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p>	<p><u>Possible misconceptions or errors:</u></p> <ul style="list-style-type: none"> • Failing to recognize that for Terrific Tunes, the cost is determined by adding the cost of a song to the previous total cost. You might ask, “What is the cost of the first song? What is the cost of the second song? How much have you paid to download 2 songs?” • Not knowing what a function is in terms of a real world context. You might ask, “What is the definition of function? What is the independent variable (or x-value) in each plan? What is the dependent variable (or y-value) in each plan? How can you determine if either plan is a function?” • Unfamiliarity with non-linear functions. Some students may think that any relation that is not linear is not a function. You might ask, “What is the definition of function? Do the plans fit the definition of function? Why or why not?” • Unfamiliarity with continuous and discrete variables. The independent variable for Super Songs is continuous. The independent variable for Terrific Tunes is discrete. Some students may think that one or the other is not a function because it consists of discrete or continuous variables. • Each plan has a different independent variable – Super Songs’ is minutes and Terrific Tunes is number of songs. Therefore the functions cannot be compared directly using the same table or same graph to determine “the best deal.” • Length of each plan. The task does not state that either plan has a time frame (i.e. monthly or yearly). 	<p><u>Possible misconceptions or errors:</u></p> <p>It is important to have students explain their thinking before assuming they are making an error or have a misconception. After listening to their explanation, ask questions that will move them toward understanding their misconception or error.</p> <ul style="list-style-type: none"> • The first song will cost \$10 and the second song will cost \$7.50, so the cost of downloading 2 songs will be \$17.50.

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<p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p>	<ul style="list-style-type: none"> • **Using your table, how can you determine if either plan represents a function? • Why can't both plans be represented by the same table? What will you need to do to compare the two plans? • **How will you know when you have found the best deal? <p><i>Constructing a graph:</i></p> <p>Ask questions such as:</p> <ul style="list-style-type: none"> • Which is the independent and which is the dependent variable for each plan? How do you know? 	<ul style="list-style-type: none"> • Students should be able to state that both plans represent functions because for every value of the independent variable (the first column in the table – either time in minutes or number of songs) there is exactly one value for the dependent variable, the cost. • Students might be tempted to put both plans on the same table. They should realize that since the independent variables are different for each plan, the same table would not make sense – 10 minutes does not mean the same things as 10 songs. • Press students to think about how they might compare the plans. Since the dependent variable is “cost” they should realize that they need to compare the costs of each plan and determine what the same cost means for each plan. They will also need to think about how the numbers of minutes and number of songs might be related. <p><i>Constructing a graph:</i></p> <p>Possible students responses:</p> <ul style="list-style-type: none"> • Students should state that the independent variables are time (for Super Songs plan) and number of songs (for Terrific Tunes.) The dependent variable is cost (for both plans.) The cost of each plan depends on either the time or number of songs.

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<p style="text-align: center;">E X P L O R E</p> <p style="text-align: center;">E X P L O R E</p>	<ul style="list-style-type: none"> • Which axes would represent which variable? • **What do you think would be sensible scales for your graph? Why? • **Using your graph, how can you determine if either plan represents a function? • Why can't both plans be represented by the same graph? What will you need to do to compare the two plans? • How will you know when you have found the best deal? 	<ul style="list-style-type: none"> • Students should state that the x-axis represents the independent variable – time in minutes for Super Songs and number of songs for Terrific Tunes. The y-axis represents the cost of the plan. • **Students should state that for Super Songs, sensible scales would begin at 0 minutes and go in increments of 10 minutes. Songs are usually close to 5 minutes long and one would probably download many songs if they were going to subscribe to a plan. For Terrific Tunes, increments of "1" would make the most sense for the scales since the cost of each song depends on the cost of the previous song. For the y-axis, the students might choose a scale of \$5 or \$10. • Students should be able to state that both plans represent functions because for every value of the independent variable (the x-axis) there is exactly one value for the dependent variable, the y-axis. If students state that they both pass the vertical line test, ask them to explain what the vertical line test means. They should be able to say that the vertical line test allows you to see if for a particular value of the independent variable, the graph passes through more than one value of the dependent variable. If so, then the graph does not represent a function. • Students might be tempted to put both plans on the same graph. They should realize that since the independent variables are different for each plan, the same graph would not make sense – 10 minutes does not mean the same things as 10 songs. • Press students to think about how they might compare the plans. Since the dependent variable is "cost" they should realize that they need to compare the costs of each plan and determine what the same cost means for each plan. They will also need to make an assumption about the approximate length of a song so that they can link number of songs to number of minutes.

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<p style="text-align: center;">S H A R E</p> <p style="text-align: center;">D I S C U S S</p> <p style="text-align: center;">A N D</p> <p style="text-align: center;">A N A L Y Z E</p>	<p style="text-align: center;"><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE</u></p> <p><u>What solution paths will be shared, in what order, and why?</u></p> <p>** Indicates responses that get at the key mathematical ideas in terms of the goals of the lesson.</p> <p>Possible Solutions to be Shared</p> <p>You might begin the discussion by having a table and graph of each plan displayed for the whole class or have students look at their own tables and graphs. Then ask the question:</p> <ul style="list-style-type: none"> • How are the plans the same and how are they different? Ask students to fill out a “same/different” chart. When students have had time to do this in their groups, build on their responses to the question to get at the mathematical goals. • **So how are the plans the same? When students respond that both plans are functions, ask them to come up and explain using the tables and graphs. How do you see that in the tables? How do you see that in the graphs? 	<p style="text-align: center;"><u>FACILITATING THE SHARE, DISCUSS, AND ANALYZE</u></p> <p><u>What solution paths will be shared, in what order, and why?</u></p> <p>The purpose of the discussion is to assist the teacher in making certain that the goals of the lesson are achieved by students. Questions and discussions should focus on the important mathematics and processes that were identified for the lesson.</p> <p>** Indicates responses that get at the key mathematical ideas in terms of the goals of the lesson.</p> <p>Possible Solutions to be Shared</p> <p>This task does not lend itself to multiple solutions as many of the previous tasks have. Instead of orchestrating the discussion around various solution paths, use the question How are the plans the same and how are they different?</p> <p>This question could serve to bring out the key mathematical ideas of the lesson within the context of the problem by using students’ responses as a starting point.</p> <p>** This question should lead to a discussion about the concept of function. Listen for a response dealing with the fact that both plans represent functional relationships.</p> <ul style="list-style-type: none"> • Students should state that both plans represent functions because for every value of the independent variable (the first column in the table and the values on the x-axis of the graph – either time in minutes or number of songs) there is exactly one value for the dependent variable, the cost.

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S H A R E D I S C U S S A N D A N A L Y Z E	<ul style="list-style-type: none"> ** How are the plans different? <p><i>Different values for the domain</i></p> <ul style="list-style-type: none"> **When a student says that the plans are based on different things you might ask what they are each based on. Then follow up with What are the possible number of minutes? What are the possible number of songs? Press students to use correct vocabulary such as independent variable, dependent variable, rational numbers, integers, etc. <ul style="list-style-type: none"> **After discussing the possible values, you might say In mathematics, we use the term, “domain,” to describe the possible values that the independent variable can have. Where do you see the domain in the table? Where do you see the domain in the graph? **Do both functions have the same domain? Why or why not? You might want to discuss the concept of “continuous” and “discrete” variables. 	<p>**There should be a variety of responses to this question. Following would be possible responses that could lead to a discussion of the goals of the lesson.</p> <p><i>Different values for the domain</i></p> <ul style="list-style-type: none"> Students might say that one plan is based on the number of minutes and the other is based on the number of songs. Students should state that for the Super Songs plan, you needed to know the number of minutes to determine the cost. The possible set of values for the independent variable, number of minutes, is any <u>rational</u> number greater than or equal to 0 since fractional minutes of a song can be downloaded and any rational number can be represented by a fraction of two integers, the denominator of which cannot be 0. Students should state that you needed to know the number of songs downloaded for the Terrific Tunes plan to determine the cost. The possible set of values for the independent variable, number of songs, is the set of whole numbers (or 0 and positive integers) because you are talking about the number of songs downloaded. You cannot download part or a fraction of a song. **Students should say that some of the domain values are in the first column of the tables but it would not be possible to display all of the domain values. They should also say that you can identify the domain in the graph by looking at the values for the x-axis. Students should state that each plan has a different domain because the independent variables represent different things.

Phase	TEACHER PEDAGOGY	STUDENT RESPONSES AND RATIONALE FOR PEDAGOGY
S H A R E D I S C U S A N D A N A L Y Z E	<p><i>Different values for the range</i></p> <p>If students do not bring up the idea that the cost structures are different for each plan, ask them what they think will happen as they continue to download more music. You might have a table prepared in advance that goes beyond 30 songs for Terrific Tunes to show how the cost levels off.</p> <p>** What values can the cost have for each plan?</p> <ul style="list-style-type: none"> • ** After discussing the possible values for each plan, you might say <i>In mathematics, we use the term, “range,” to describe the possible values that the dependent variable can have. Where do you see those values in the table? In the graph?</i> • **Why does one plan have a range that goes higher the more music that is downloaded while the other has a limited range? • You might challenge students to try to find out the number of songs they will need to download to reach this cost (due to rounding.) 	<p><i>Different values for the range</i></p> <ul style="list-style-type: none"> • Students might notice that the cost of the Super Songs plan continues to increase while the cost of the Terrific Tunes plan levels off at \$40.00 (due to rounding). Students should state that the cost for the Super Songs plan goes from \$0.00 to infinity. The more minutes that music is downloaded, the more the cost increases. Students may be tempted to say that the cost has integer or whole number values depending on the intervals they used in their tables. If they do, ask them to give you the cost of downloading 32.4 minutes of music. The cost of the Terrific Tunes plan, on the other hand, goes from \$0.00 to \$40. • Students should state that you can see the values in the second column of the table. Students should state that you can see the values in the graph by determining, if possible, the lowest and highest points on the graph. • Students should state that the cost for Super Songs continues to increase as you increase the number of minutes. Since you could theoretically download an infinite number of minutes, the cost would increase infinitely. However; for Terrific Tunes, the cost depends on the cost of the previous song. Since each song costs $\frac{3}{4}$ of the previous song, its cost will be less than the previous song. By continuously taking $\frac{3}{4}$ of smaller and smaller numbers, the cost of a song could eventually reach 0. That means that at a certain point, there would be no additional cost to download a song. For this plan, the highest cost is \$40.

