SURROUND THE POOL LESSON – Algebra 1

Overview: In this lesson, students write expressions for the number of one-foot-square tiles needed to surround a square swimming pool of unknown dimensions. Students should be encouraged to write as many different expressions as possible. The equivalence of the expressions should be discussed by evaluating the expressions for several different values and by simplifying the expressions.

Goals:
1. use variables to write algebraic expressions for a relationship
2. determine equivalence of algebraic expressions
3. evaluate algebraic expressions

Algebra Standards:

AF 1.1   Use variables and appropriate operations to write an expression, (an equation, an inequality, or a system of equations or inequalities) which represents a verbal description

AF 1.2   Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$

AF 1.3   Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

AF 1.4   Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

Mathematical Reasoning Standards:

MR 2.5   Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

MR 2.6   Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

Building on Prior Knowledge:

AF 1.0   Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions.

AF 3.0   Students investigate geometric patterns and describe them algebraically

Materials: Surround the Pool task (attached), graph paper, square tiles, calculators, chart paper, markers

Phase	Action	Comments
S E T U P	Solve the task yourself prior to the lesson.	It is critical that you solve the problem in as many ways as possible in preparation for the lesson so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different strategies for solving the problem will be presented.
	SETTING UP THE LESSON Ask: **Think about the hottest day of the summer. If you had a swimming pool in your back yard, what would it look like? Would you want to step out of the pool onto the grass?** Ask for a volunteer to read the problem while the remaining students follow along. Then have several students explain to the class what they are trying to find when solving the problem.	SETTING UP THE LESSON As students describe the task, make certain they indicate that the goal is to find as many expressions as possible for describing the number of tiles surrounding the pool. In addition, make sure students understand that they are trying to find the number of tiles needed for the border of *any* pool, not just one specific pool. Be careful not to tell the students how to solve the task since your goal is for students to do the problem solving.

Phase	Action	Comments
E X P L O R E	PRIVATE PROBLEM-SOLVING TIME Give students 5 - 7 minutes of private think time to begin to solve the problem individually before beginning work in their small groups.	PRIVATE PROBLEM-SOLVING TIME Make sure that students' thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes. During this time watch for students who are having trouble getting started or staying on task.
	FACILITATING SMALL-GROUP PROBLEM SOLVING Circulate among the groups as they work providing assistance as needed and asking questions to assess students' understanding of mathematical ideas.	FACILITATING SMALL-GROUP PROBLEM SOLVING As students work in small groups, you have the opportunity to listen to what they are doing and saying. This will give you valuable insights into what students understand and what they are struggling with. Do not feel that you have to talk when you visit a group unless assistance is needed or you want to press students to explain some aspect of their work that is unclear, incomplete, or incorrect.
	What do I do if students have difficulty getting started? Assist students/groups who are struggling to get started by prompting with such questions as: - **What do you know about the pool?** - **How many tiles would you need if it was a small pool, 3 ft. by 3ft.? 4 ft. by 4ft.?** - **Could you draw the pool and tiles or build it with the square tiles?**	*What do I do if students have difficulty getting started?* By asking, "What do you know about the hot tub?" you are providing students with a way to focus on what they know, what they are given, and what they need to find. If students learn to ask this regularly, they will have a means of getting started when problem solving.

Which problem-solving strategy might students use? How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will approach the problem using a variety of strategies. A subset of possible solutions and strategies are listed below with accompanying questions that could be asked to assess students' understanding and to advance their learning.

Be sure to ask students to relate their explanations to the picture and to the context (e.g., the problem is asking students to find the number of tiles needed to build a boarder. It is not asking for the dimensions or the perimeter of the boarder).

$4s + 4$ or $s + s + s + s + 4$

♦ **What does the "s" represent? What mathematical name do we use for "s"? What do we call the 4 in front of the "s"? What do we call the 4 after the plus sign?**
♦ **Where do you see each term in the expression found in the picture?**
♦ **Why do you have to take 4 times s? Why must you add on 4?**
♦ **Suppose we had a square swimming pool whose sides were 89 ft. Describe in words and show on paper how you would find how many tiles would be needed using your expression.**

If students have also created other expressions, ask: **How does this expression connect to your other expression?**

Which problem-solving strategy might students use? How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

The teacher's role when students are working in small groups is to circulate so as to monitor students as they work and to ask questions that assess and advance their learning.

Students are required to write an algebraic expression to solve this problem. Students should be able to identify the variable, the coefficient, and the constant in any of the expressions.

Students should also learn to determine the equivalence of the expressions, to connect the expressions to a diagram, and to talk about the relationship between the expressions. Doing this prior to formally introducing them to procedures such as applying the commutative, associative, and distributive properties and combining like terms, will help them learn these ideas in meaningful ways.

As students work in their groups, they will be working toward mathematical reasoning standards 2.5 and 2.6 as described below. You should see students solving the problem in different ways and talking about their solutions in their groups. They should be able to explain why their solution is correct.

MR 2.5 – *Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.*

MR 2.6 – *Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.*

$4s + 4$

Students should state the "s" is the number of one-foot tiles that will fit along the edge of the pool (it is also the length of the side of the pool) and it is called a variable. The 4 in front of "s" is called the coefficient and it indicates the pool has 4 sides each of which will contain "s" tiles. The "+4" is a constant and represents the 4 square tiles, 1 in each corner.

Students should state that you would multiply 4 times 89 and then add 4. They should show $4(89) + 4$ and get an answer of 360 tiles. They should be able to relate this explanation to a diagram of the pool and its border.
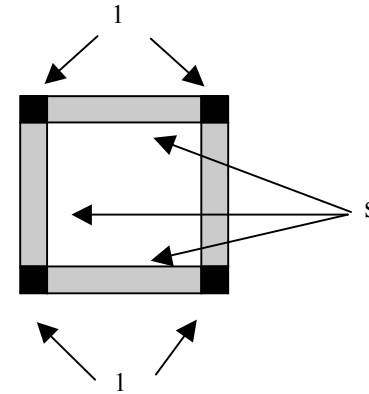
4(s + 1)

◆ **What does the "s+1" represent? What mathematical name do we have for "s"? What do we call the 4 in front of the parentheses? Why is s+1 in parentheses?**
◆ **Where do you see "s + 1" in your diagram?**
◆ **Why do you have to take 4 times s + 1?**
◆ **Suppose we had a square swimming pool whose sides were 89 ft. Describe in words and show on paper how you would find how many tiles would be needed using your expression.**
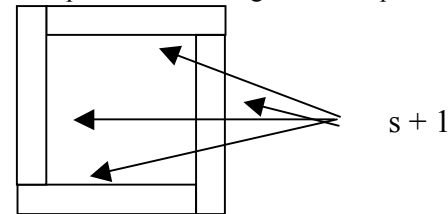
If students have also created other expressions, ask: **How does this expression connect to your other expression?**

4(s+1)

Students should state the "s + 1" is the number of one foot tiles that will fit along one side of the pool plus one to fill in one of the corners of the border and "s" is called a variable. The 4 in front of "s+1" is a coefficient and it indicates the border is made up of 4 sets each which has "s+1" tiles. The s+1 is in parentheses because it represents number of tiles in part of the border and to find the number of tiles in the entire border you must multiply by 4.

Students should state that you would first add 89 + 1 and take that answer times 4. They should show 4(89 + 1) and get an answer of 360 tiles. They should be able to relate this explanation to a diagram of the pool and its border.

2(s + 2) + 2s

◆ **What does the "s+2" represent? What mathematical name do we have for "s"? What do we call the 2 in front of the parentheses? Why is s+2 in parentheses?**
◆ **Where do you see "s + 2" in your diagram? Where do you see 2s in your diagram?**
◆ **Why do you have to take 2 times s + 2?**
◆ **Suppose we had a square swimming pool whose sides were 89 ft. Describe in words and show on paper how you would find how many tiles would be needed using your expression.**

If students have also created other expressions, ask: **How does this**

2(s+2)+2s

Students should state the "s + 2" is the number of one foot tiles that form the border on the top and bottom of the pool and "s" is called a variable. The 2 in front of "s+2" is a coefficient and it indicates that both the top and bottom border have the same number of tiles. The s+2 is in parentheses because it represents the number of tiles in the top and bottom borders and therefore must be multiplied by 2. The "2s" represents the number of tiles in the border on the remaining sides of the pool, each of which has s tiles.

Students should state that you would first add 89 + 2 and then multiply that answer times 2. Next you would take 2 times 89 and then add the two answers. They should show 2(89+2) + 2(89) which is 182 + 178 and get an answer of
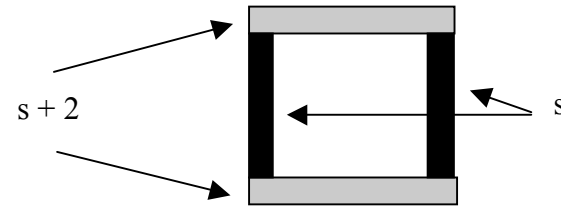
expression connect to your other expression?

360 tiles. They should be able to relate this explanation to a diagram of the pool and its border.

$4(s + 2) - 4$

$4(s+2)-4$

- ♦ **What does the "s+2" represent? What mathematical name do we have for "s"? What do we call the 4 in front of the parentheses? Why is s+2 in parentheses?**
- ♦ **Where do you see "s + 2" in your diagram? Why must you subtract 4? Where do you see this in your diagram?**
- ♦ **Why do you have to take 4 times s + 2?**
- ♦ **Suppose we had a square swimming pool whose sides were 89 ft. Describe in words and show on paper how you would find how many tiles would be needed using your expression.**

If students have also created other expressions, ask: **How does this expression connect to your other expression?**

Students should state the "s + 2" is the number of tiles that form the border on the top and bottom of the pool and "s" is called a variable. The 4 in front of "s+2" is a coefficient and it indicates that each of the four outer sides of the boarder contains "s+2" tiles. The s+2 is in parentheses because it represents the number of tiles on one side of the boarder and that to find the number of tiles in the entire boarder you must be multiply by 4. Then 4 must be subtracted because you double counted all the tiles in the corners.

Students should state that you would first add $89 + 2$ and multiply the product by 4. You would then subtract 4. They should show $4(89 + 2) - 4$ and get an answer of 360 tiles. They should be able to relate this explanation to a diagram of the pool and its border.
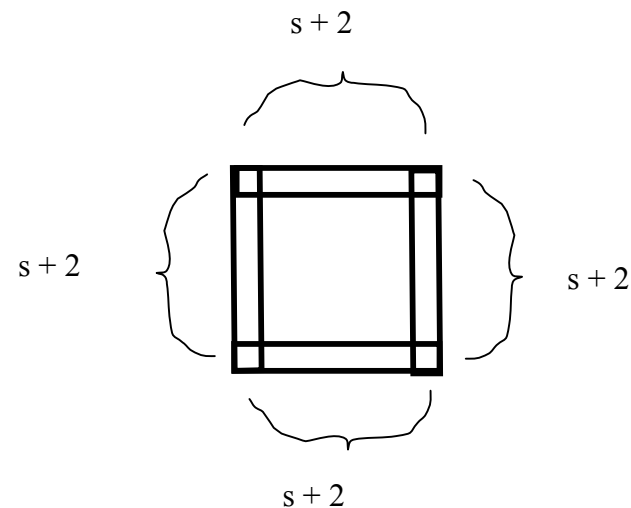
FACILITATING THE GROUP DISCUSSION

In what order will I have students post solution paths so I will be able to help students make connections between the solution paths?
Ask students to explain their solutions to you as you circulate among the small groups. Look for solutions that will be shared with the whole group and consider the order in which they will be shared. You might want to have the least complex expressions shared first: 4s + 4 followed by 4(s+1). Then have the more complex expression shared: 2(s+2) + 2s and 4(s+2) – 4.

If no one has come up with one or more of the expressions listed above, you might want to introduce a new expression and ask: **What about this expression? Does it describe how many tiles would be needed to surround the pool?**

As each new group shares their approach, write each new expression on the board, overhead, or on newsprint and list the name of student(s) who produced it.

The goal is to discuss mathematical ideas associated with algebraic expressions including writing, simplifying, and evaluating expressions.

Begin by having students share their solutions. Ask questions similar to those in the "EXPLORE" phase above for each representation.

In general, ask each student presenter:

♦ **Can you explain your expression using the context of the problem?**
♦ **Can you show how your expression fits with the picture?**
♦ **What does the "s" represent? What mathematical name do we use for "s"?**
♦ **What number is the coefficient and what does the coefficient mean in your expression?**
♦ **What purpose do the parentheses serve?**
♦ **What is "constant" and why doesn't this ever change?**

As each new representation is given, connect it to the other representations. Ask: **How is your solution the same as or different from the other solutions?**

After all students have presented, ask: **So if we had a square**

FACILITATING THE GROUP DISCUSSION

In what order will I have students post solution paths so I will be able to help students make connections between the solution paths?
Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group.

Try to use the following "accountable talk" moves as you facilitate the discussion. This will help ensure that all students are actively engaged in making sense of the solutions begin presented.

♦ As students are explaining, ask them to point to the part of the expression or the diagram so that others can follow.
♦ Ask students to put another student's explanation in their own words.
♦ Ask students to agree or disagree with a student's statement and then provide an explanation of WHY they agree or disagree.

The goal here is to explicitly address the standards toward which you are working (AF 1.1, 1.2, 1.3, 1.4 and MR 2.5 and 2.6). Here is a summary of what students need to be able to do in order to be working on the specific standard.

MR 2.5 – explain their expressions and their diagrams and how they are connected.

MR 2.6 – use correct terminology and notation and answer questions about their expressions and diagrams.

AF 1.1 – correctly use variables to write an expression.

AF 1.2 – correctly evaluate the expression for a pool that is 80 x 89.

AF 1.3 – correctly simplify the expressions.

AF 1.4 – identify the variable, coefficient, and constant in an expression.

By realizing the equivalence of the expressions, students can begin to

swimming pool 89 ft. by 89 ft., how many tiles would we need to surround it? Let's use each expression to see what we get.

Students should notice that they get the same answer no matter which expression is used.

♦ **What does that mean if all of the expressions gave us the same answer?**
♦ **How can all of these expressions be the same? How could we show that 4s + 4 is the same as or equivalent to 4(s +1) ?**
♦ **What does the coefficient of 4 in the expression 4(s+1) mean mathematically?**

understand the distributive property.

For example, with the express 4(s+1) you could write out the 4 groups, either vertically or horizontally and have the students simplify the expression so that they get 4s + 4.

(s+1) + (s+1) + (s+1) + (s+1) = s + s + s + s + 1 + 1 + 1 + 1 = 4s + 4

Do the same for the other expressions to show they are also equivalent to 4s + 4.

These types of questions begin to address standard *AF 1.3 – Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.*

HOMEWORK

Determine if the following expressions are equivalent. Explain how you made your decision.

1. 3(x + 2) and 3x + 2
2. 2(a + 1) and 2a + 2
3. 3(b + 2) – 3 and 3b + 3

HOMEWORK

This homework assignment will provide students with an opportunity to apply the distributive property, to explore equivalence, and to explain their thinking.

Surround the Pool

SURROUND THE POOL
Adapted from <u>Navigating through Algebra in Grades 6 – 8</u>, NCTM

Hot tubs and in-ground swimming pools are sometimes surrounded by borders of tiles. This drawing shows a square swimming pool surrounded by a border of square tiles. Each tile measures 1 foot on each side.
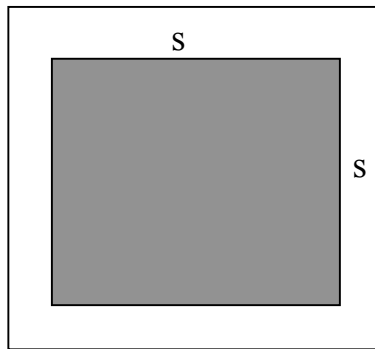
1. Write an expression describing the total number of 1-foot square tiles needed for the border of the pool. Show how your expression can be seen in the drawing.

2. Solve the problem again using a different expression and explain how the second expression can be seen in the drawing.
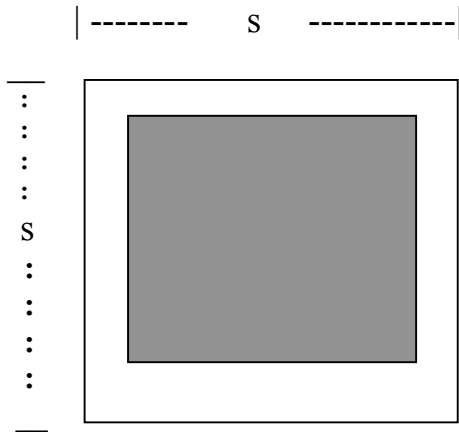
3. Show that the expressions are equivalent.

4. Use a diagram to show the difference between the expressions 4x + 1 and 4(s + 1).

This drawing shows a square swimming pool surrounded by a border of square tiles. Each tile measures 1 foot on each side.

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1. Write an expression describing the total number of 1-foot square tiles needed for the border of the pool. Show how your expression can be seen in the drawing.

2. Solve the problem again using a different expression and explain how the second expression can be seen in the drawing.

3. Show that the expressions are equivalent.