Overview:

In this lesson students write the general expression for a pattern that is growing quadratically. They use that general expression to determine the number of tiles in any pattern in the sequence. Students then compare various equivalent quadratic expressions and simplify them to demonstrate that they are algebraically equivalent. This requires students to make connections between and among the expressions and the picture or table representation of the relationship of the place of the pattern in the sequence and the number of tiles in the pattern; and to evaluate and factor quadratic expressions.

CA Standards Addressed:

10.0 Students add, subtract, multiply and divide monomials and polynomials. Students solve multi-step problems, including word problems, using these techniques.

14.0 Students solve a quadratic equation by factoring or completing the square.

23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

24.0-25.0 The reasoning standards are in italics within the algebra standards. When mathematical reasoning is expected in the lesson the text will be labeled Mathematical Reasoning within the text.

Mathematical Goals of the Lesson:

Students will determine the expression for the number of tiles in any pattern in the sequence.
Students will interpret quadratic expressions in terms of the problem.
Students will solve the problem using a variety of strategies
Students will demonstrate the equivalence of various quadratic expressions by adding, subtracting, multiplying or dividing monomials and polynomials.
Students will justify their solutions to the problem.
**Access Strategies**: Throughout the document you will see icons calling out use of the access strategies for English Learners, Standard English Learners, and Students With Disabilities.

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<tr>
<th>Access Strategy</th>
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<tbody>
<tr>
<td>Cooperative and Communal Learning Environments</td>
<td><img src="image" alt="CLE" /></td>
<td>Supportive learning environments that motivate students to engage more with learning and that promote language acquisition through meaningful interactions and positive learning experiences to achieve an instructional goal. Working collaboratively in small groups, students learn faster and more efficiently, have greater retention of concepts, and feel positive about their learning.</td>
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<tr>
<td>Instructional Conversations</td>
<td><img src="image" alt="IC" /></td>
<td>Discussion-based lessons carried out with the assistance of more competent others who help students arrive at a deeper understanding of academic content. ICs provide opportunities for students to use language in interactions that promote analysis, reflection, and critical thinking. These classroom interactions create opportunities for students' conceptual and linguistic development by making connections between academic content, students' prior knowledge, and cultural experiences.</td>
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<tr>
<td>Academic Language Development</td>
<td><img src="image" alt="ALD" /></td>
<td>The teaching of specialized language, vocabulary, grammar, structures, patterns, and features that occur with high frequency in academic texts and discourse. ALD builds on the conceptual knowledge and vocabulary students bring from their home and community environments. Academic language proficiency is a prerequisite skill that aids comprehension and prepares students to effectively communicate in different academic areas.</td>
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<tr>
<td>Advanced Graphic Organizers</td>
<td><img src="image" alt="GO" /></td>
<td>Visual tools and representations of information that show the structure of concepts and the relationships between ideas to support critical thinking processes. Their effective use promotes active learning that helps students construct knowledge, organize thinking, visualize abstract concepts, and gain a clearer understanding of instructional material.</td>
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Academic Language Goals of the Lesson:

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<th>Academic Language:</th>
<th>Materials:</th>
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<tr>
<td>Students express quantitative relationships by using algebraic terminology, expressions, and equations. Use variables and appropriate operations to write an expression and an equation. Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$. Simplify numerical expressions by applying properties of rational numbers (e.g., distributive, associative, commutative) Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.</td>
<td>Quadratics</td>
<td>S-pattern task (attached); square tiles</td>
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<tr>
<td></td>
<td>Quadratic equation</td>
<td>Four Fold Recording Sheet</td>
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<tr>
<td></td>
<td>Pattern</td>
<td>Straight-edge</td>
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<td></td>
<td>Equivalent quadratic expressions</td>
<td>Pencil</td>
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<td></td>
<td>Factor quadratic expressions</td>
<td>Paper</td>
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Assumption of Prior Knowledge:

Students express quantitative relationships by using algebraic terminology, expressions, and equations. Use variables and appropriate operations to write an expression and an equation. Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$. Simplify numerical expressions by applying properties of rational numbers (e.g., distributive, associative, commutative) Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

Academic Language:

- Quadratics
- Quadratic equation
- Pattern
- Equivalent quadratic expressions
- Factor quadratic expressions

Materials:

- S-pattern task (attached); square tiles
- Four Fold Recording Sheet
- Straight-edge
- Pencil
- Paper

Connections to the LAUSD Algebra 1, Unit ____ , Instructional Guide

Understand Operations on Polynomials

2.0, 10.0

- Perform operations on monomials and polynomials

Understand factoring of Polynomials

11.0, 14.0

- Factor 2nd degree polynomials over the integers
- Use the zero-product rule and factoring as well as completing the square to solve simple quadratics
**Key:**  
Suggested teacher questions are shown in bold print.  
Questions and strategies that support ELLs are underlined identified by an asterisk.  
Possible student responses are shown in italics

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<tr>
<th>Phase</th>
<th>SET UP PHASE: Setting Up the Mathematical Task—Part 1</th>
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<tr>
<td><strong>SET UP</strong></td>
<td><strong>INTRODUCING THE TASK</strong></td>
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<td>S</td>
<td><strong>Prior to the lesson:</strong></td>
</tr>
<tr>
<td>E</td>
<td>• arrange the desks so that students are in groups of 4.</td>
</tr>
<tr>
<td>T</td>
<td>• determine student groups prior to the lesson so that students who complement each other’s skills and knowledge core are working together.</td>
</tr>
<tr>
<td>U</td>
<td>• place materials for the task at each grouping.</td>
</tr>
<tr>
<td>P</td>
<td>• solve the task yourself.</td>
</tr>
</tbody>
</table>

**HOW DO I SET-UP THE LESSON?**  
Ask students to follow along as you read the problem. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem.

Clarify any confusions students may have but do not suggest specific values for their investigation.

- Give the students individual time to think about this question and begin a class discussion
- *A Frayer model could be used to assist students in developing a clear understanding of some of the terms to be used in the lesson.*
- *If the students are NOT able to easily answer the first question then it may be appropriate (especially for EL, SEL and SWD) to spend time doing an additional hands on activity (simple linear pattern) to reinforce concept.*

To assist ELLs’ participation in the class discussion*:
- Allow time for students to first talk in small groups (pairs) and then have the groups report to the whole class.*
- Reinforce appropriate language as students communicate their ideas (e.g. re-voice a student’s contribution in complete, grammatically correct language). Ask students if you have captured what they said*.
- Create work groups that are heterogeneous according to language proficiency.*
- Model appropriate mathematical language, emphasizing vocabulary used in appropriate context.*
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<th>Phase</th>
<th>EXPLORE PHASE: Supporting Students’ Exploration of the Task</th>
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<td>STRUCTURE</td>
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<tr>
<td>EXPLORE Phase</td>
<td>PRIVATE THINK TIME</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>Give students 5 - 7 minutes of private think time to begin solve the problem individually. Circulate among the groups assessing students’ understanding of the idea below.</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>FACILITATING SMALL GROUP PROBLEM SOLVING</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>• After about 15 minutes, ask students to work with their partner or in their small groups to discuss what they discovered.</td>
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<tr>
<td>EXPLORE Phase</td>
<td>• As you circulate among the groups, press students to come up with more than one solution and then show that the expressions are equivalent. After explaining their initial solutions you might say “Now look at the figures again.” Find another way to look at how the pattern is growing from one figure to the next.</td>
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<td>EXPLORE Phase</td>
<td>After there is more than one solution in the group press student to show that the expressions are equivalent algebraically by using their prior knowledge of various properties such as the distributive property, properties of exponents, and rules for adding and subtracting like terms.</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>PRIVATE THINK TIME</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>Make sure that students’ thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes.</td>
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<td>EXPLORE Phase</td>
<td>FACILITATING SMALL GROUP PROBLEM SOLVING</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>The teacher’s role when students are working in small groups is to circulate and listen with the goal of understanding students’ ideas and asking questions that will advance student work.</td>
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<tr>
<td>EXPLORE Phase</td>
<td>• Be persistent in asking questions related to the mathematical ideas (see question suggestions in the following section), exploration strategies, connections between representations.</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>• Be persistent in asking students to explain their thinking and reasoning.</td>
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<tr>
<td>EXPLORE Phase</td>
<td>• Be persistent in asking students to explain, in their own words, what other students have said.</td>
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<tr>
<td>EXPLORE Phase</td>
<td>• Be persistent in asking students to use appropriate mathematical language.</td>
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<tr>
<td>EXPLORE Phase</td>
<td>What do I do if students have difficulty getting started?</td>
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<tr>
<td>EXPLORE Phase</td>
<td>By asking a question such as “What do you notice about the figures pattern?” the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine.</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>What misconceptions might students have?</td>
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<tr>
<td>EXPLORE Phase</td>
<td>Misconceptions are common. Students may have learned the information incorrectly or they may generalize ideas prematurely. Some strategies for helping students discover when they have made an error include:</td>
</tr>
<tr>
<td>EXPLORE Phase</td>
<td>- Ask students to extend a pattern and compare it to what they predicted pattern should be.</td>
</tr>
</tbody>
</table>
• How are the figures changing from one to the next?
  - Give students square tiles and ask them to construct the 1st, 2nd, and 3rd figures. Then ask: How did you know how to construct the figures? How would you construct the 4th figure?

  What misconceptions might students have?
  Look for and clarify any misconceptions students may have.

  a. looking only at the first two figures and assuming it’s a linear growth pattern. How many tiles are in the third figure? The fourth figure? Does this match your pattern?

Which problem-solving strategies might be used by students?
How do I advance students’ understanding of mathematical concepts or strategies when they are working with each strategy?

Students will approach the problem using a variety of strategies. Some strategies are shown below. Questions for assessing understanding and advancing student learning are listed for each.

Using the drawing

A. moving a row and making it a column to form a square plus one tile

   How do the dimensions of the square compare with the figure number? What would the dimensions for the 100th square be? How would you draw the nth figure? How would you write an expression for the nth figure?

Which problem-solving strategies might be used by students?
How do I advance students’ understanding of mathematical concepts or strategies when they are working with each strategy?

Using the drawing

A. T = (N • N) + 1 OR N^2 + 1

Figure 5 Move the bottom row and make it a column resulting in a 5 by 5 square plus one tile

Figure N An N by N square plus one tile

B. T = (N + 1)(N - 1) + 2

Figure 5 6 by 4 rectangle + 2 tiles
B. -seeing each figure as a rectangle with 2 additional tiles at opposite corners

How do the dimensions of the rectangle compare with the figure number? What would the dimensions for the 100th rectangle be? How would you draw the nth figure? How would you write an expression for the dimensions of the nth rectangle? The nth figure?

Simplify your expression.
Press students to use the properties and rules they have learned dealing with monomials and polynomials.

Ask them to connect the expression $N^2 + 1$ to the diagram.

C. -seeing each figure as a square with a row of tiles 1 larger than the side of a square above the square and another row below the square

How do the dimensions of the square compare with the figure number? What would the dimensions for the 100th square be? What would the rows for the 100th figure look like? How would you draw the nth figure? How would you write an expression for the nth square? The nth figure?

Simplify your expression.

Students may use the “FOIL” method to arrive at $N^2 - N - N + 1 + 2N$ and then rules for combining like terms to simplify the expression to $N^2 + 1$. You might ask Why does the FOIL method work? How does it connect to the distributive property? How did you know which terms to combine with each other?

Ask them to connect the expression $N^2 + 1$ to the diagram.

D. $T = N(N+1) - (N-1)$.

- Figure 5 a 5 by 6 rectangle with 4 tiles removed
- Figure N an N by (N+1) rectangle with N-1 tiles removed
D. -moving the corner square tile to the beginning of its row and then seeing the figure as a rectangle from which tiles must be subtracted

How do the dimensions of the rectangle compare with the figure number? What would the dimensions for the 100th rectangle be? How would you draw the nth figure? How would you write an expression for the dimensions of the nth rectangle? The nth figure?

Simplify your expression.

Press students to use the properties and rules they have learned dealing with monomials and polynomials.

Students should use the distributive property to arrive at \( N^2 + N - (N - 1) \) and then the distributive property again to arrive at \( N^2 + N - N + 1 \). They should then use rules for combining like terms to simplify the expression to \( N^2 + 1 \). You might ask How does the distributive property work?

Ask them to connect the expression \( N^2 + 1 \) to the diagram.

Using a table

- student constructs a table by counting the tiles

What pattern do you notice in the number of tiles? If students have previous experience with linear growth patterns, they may attempt to find a linear relationship. You could ask them How is the number of tiles changing in the table?

Press students to make the connection to a quadratic pattern. Then press them to connect the expression to the diagram. You might ask So how can you see this pattern in the figures?

MONITORING STUDENTS’ RESPONSES

As you circulate, attend to students’ mathematical thinking and to their conjectures, in order to identify those responses that will be shared during the Share, Discuss, and Analyze Phases.
Sharing, Discussing, and Analyzing

Orchestrating the mathematical discussion: a possible Sequence for sharing student work, Key Questions to achieve the goals of the lesson, and possible Student Responses that demonstrate understanding.

Revisiting the Mathematical Goals of the Lesson:

The purpose of this sharing/discussion is to make explicit the conclusions the exploration.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Sequencing of Student Work and Possible Questions</th>
<th>Rationale and Mathematical Ideas</th>
<th>Student Responses and Comments</th>
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<tr>
<td>SHARE DISCUSS ANALYZE</td>
<td>FACILITATING THE GROUP DISCUSSION</td>
<td>All students should develop a rich understanding of the mathematical ideas associated with the quadratic expressions and their equivalence.</td>
<td>FACILITATING THE GROUP DISCUSSION</td>
</tr>
<tr>
<td></td>
<td>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</td>
<td>Students should be able to make connections between different solution paths.</td>
<td>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</td>
</tr>
<tr>
<td></td>
<td>As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of the diagram.</td>
<td></td>
<td>Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group. For this particular problem it could be best to have the solution N2 + 1 shared first so that all other expressions can be simplified to that. If no one has come up with that solution you could say This is the solution I came up with. Then demonstrate the solution’s connection to the diagram. Is my expression equivalent to yours? Show me.</td>
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<td>The work of at least two groups should be chosen. Look for a variety of strategies used by the students to develop and write their rules, such as “Newton’s difference” approach, and the once shown above in the Explore phase.</td>
<td></td>
<td>Recognizing equivalent forms of expressions and being able to convert flexibly among them means that a student should be able to write a polynomial in factored form. That is, a student should understand that x2 + 7x + 10 = (x + 2)(x + 5). Further, students should recognize that both expressions represent a quadratic function that crosses the x-intercept at (2, 0) and (-5, 0). This should become evident once they begin to graph them.</td>
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<td></td>
<td>If the work can be displayed, ask students to post them in the front of the classroom, then have the class look at the students’ work, without any explanation, and ask students other than those in the contributing groups how they think the problem has been solved. The goal is to discuss mathematical ideas associated with the quadratic expressions and their equivalence.</td>
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<td>If it is not possible to display the work to the whole class, have groups present and explain their own</td>
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</table>
What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? (Driving Questions *)

N^2 + 1 solution
If students have come up with the solution, N^2 + 1, ask them to present their solution and explain how their solution connects to the diagram. You might ask What does the N^2 look like in the diagram? Where do you see the “1”?

Ask students who have come up with a different solution to present their solutions:

T = (N + 1)(N - 1) + 2 solution
Ask students to present their solution and explain how it connects to the diagram.

Driving Question
* How is your solution equivalent to N^2 + 1? Show us.
Make certain to press on the same questions noted above for this solution.

T = (N-1)(N-1) + 2N solution
Ask students to present their solution and explain how it connects to the diagram.

Driving Question
* How is your solution equivalent to N^2 + 1? Show us.
Make certain to press on the same questions noted above for this solution.

T = N(N+1) – (N-1) solution
Ask students to present their solution and explain how it connects to the diagram.

Driving Question
* How is your solution equivalent to N^2 + 1? Show us.
Make certain to press on the same questions noted above for this solution.

Table Solution
Ask students to demonstrate how they arrived at the
table and explain how it connects to the diagram.

**HOMEWORK**
You could give the Extend Pattern of Tiles problem as a homework assignment.

**Combining like terms** – Students should explain that only terms that are the same can be added or subtracted. For example, N2 does not mean the same thing as N and therefore the two cannot be combined.

**HOMEWORK**
In addition, students could work on problems like the one below to make connections to real life applications of factoring polynomials.

Measure the length and width of a rectangular room in your home, to the nearest foot. Suppose you want to buy a carpet to fit in the room with a space \( x \) feet wide on all four sides. Find a model for the area of the rug. Write it as a quadratic trinomial. Use the model to find the cost of the carpet if \( x = 3 \) and carpet costs $7.50 a square foot.