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Grade 4 Quarter Two  
Concept Lesson Professional Development  
“Multi-Digit Division”

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Los Angeles Unified School District  
Elementary Mathematics  
Fourth Grade  
August 9 & 10, 2006

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# Outcomes for the Day

- Explore the content involved in the Fourth Grade Quarter 2 Concept Lesson
- Revisit Thinking Through a Lesson Protocol (TTLP)
- Engage in the Fourth Grade Quarter 2 Concept Lesson
- Explore the teacher practices modeled in the Concept Lesson through the use of the TTLP
- Examine Student Misconceptions and Questions to Address Them

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# Outcomes for the Morning

- Explore and understand different strategies for solving division problems
- Discuss the two major models of division
- Investigate strategies for developing the traditional algorithm for division
- Create division word problems
- Connect our learning to Big Ideas, Concepts, and Skills

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## Warm-Up Problem: “Sid’s Jelly Beans”

Sid has a bag of 143 jelly beans. He wants to put the same number of jelly beans in 5 bags. How many jelly beans can he put in each bag?

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## Reading I:

# “Toward Computational Fluency”

- Take 20 minutes to read through the passage with the questions in mind.
- You might also record any ideas from the text that interest, puzzle, or surprise you.
- After reading, choose a facilitator within your table group. The facilitator leads the table group in discussion.
- After 20 minutes, we will summarize whole group.

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# Models of Division:

## Partitive Division (Fair Share)

- In the partitive model of division, you know
  - how many you are starting with, and
  - how many groups you will be making
  
- You are trying to find out
  - how many are in each group
  
- The action involved is “dealing out”

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# Models of Division:

## Quotative (Measurement) Division

- In the quotative (measurement) model of division, you know
  - how many you are starting with, and
  - how many are in each group
- You are trying to find out
  - how many groups there are
- The action involved is “measuring out”

## Models of Division

Determine what you know, what you are trying to find out, the action involved, and the problem type for each word problem.

Word Problem	What do you know?	What do you need to find out?	What action is suggested by the problem?	Problem Type
A box of pencils has 288 pencils in packets of 12. How many packets of pencils are there?				
There are 365 days in a year. Since there are 52 weeks in a year, how many days should there be in each week?				
A show made \$171.25. How many people attended the show if it made \$1.25 for each person who attended?				
A show made \$171.25. If 137 people attended, how much money did the show make for each person?				

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# Models of Division:

## Activity

- Work with the other members of your table group.
  - For each problem, determine...
    - What do you know? *The number of groups, or the number in each group?*
    - What are you trying to find out? *The number of groups, or the number in each group?*
    - What action is suggested by the problem? *Dealing, or measuring?*
    - What is the problem type? *Partitive Division, or Measurement Division?*
  - When you have finished the four problems, decide which question was most useful in helping you determine if the problem was partitive or measurement division.
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## Reading II:

### “Invented Strategies for Division”

- Which examples do not demonstrate direct modeling?
- Which examples demonstrate direct modeling?
- In what ways do direct modeling strategies address the needs of diverse learners such as Special Needs Students, English Language Learners, or GATE students?
- Why might these strategies need to remain student inventions and not strategies that we directly teach?

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# Developing the Traditional (Standard) Algorithm for Division

- Suggested sequence for developing the algorithm for Long Division
  - Begin with Models
  - Develop Written Records
  - Record Explicit Trades

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# Begin with Models

- Use problems with contexts that match base-10 models.
- Example:
  - 100 candies in each box (base-10 flat)
  - 10 candies in each carton (base-10 rod)
- Problem Example:
  - *We have 5 boxes, 8 cartons, and 3 pieces of candy to share with 4 schools evenly. How much candy will each school get?*

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# Develop Written Records

- Steps in the written record
  - *Share* and record the number of pieces put in each group.
  - *Record* the number of pieces shared in all. Multiply to find this number.
  - *Record* the number of pieces remaining. Subtract to find this number.
  - *Trade* (if necessary) for smaller pieces, and combine with any that are there already. Record the new total number in the next column.

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# Record Explicit Trades

- The traditional algorithm uses some short cuts that hide trades that have occurred between different place values.
- Van de Walle suggests recording trades, revealing where and how trades occur.

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# Creating Word Problems for Division (Optional, if there is time)

- Each table group creates a measurement division and partitive division word problem.
- Create a chart for each problem. Each chart should include:
  - The problem
  - Illustrations of possible student strategies
    - Try to create 3 or 4 different solution strategies
- When everyone is finished, we'll have a gallery walk to share each other's work.

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# Making Connections: Big Ideas, Concepts and Skills

- How might having students solve problems using their own sense-making strategies help them make connections to the Big Ideas, Concepts, and Skills in this quarter?
- What other Big Ideas could students make connections to?

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# Thinking Through a Lesson Protocol: Considering and Addressing Student Misconceptions and Errors

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# Outcomes

- Review Thinking Through a Lesson Protocol (TTLP)
- Engage in Fourth Grade Lesson considering components of the TTLP that the facilitator demonstrates
- Debrief the lesson with the TTLP as a frame for discussion
- Examine student responses to the Fourth Grade task and determine what the student knows and understands
- Develop questions to scaffold the learning of students who exhibit misconceptions or make errors
- Discuss the value of considering student misconceptions and errors and of developing questions to address them

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# Thinking Through a Lesson Protocol

- Review the Thinking Through a Lesson Protocol
  - What similarities does it have to previous lesson planning tools you have used?
  - What differences does it have from previous lesson planning tools you have used?
- As you engage in the fourth grade concept lesson, think about components of the TTLP that the facilitator demonstrates and/or considers as s/he engages you in the lesson.

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# Thinking Through a Lesson Protocol

- Revisit the TTLP and identify those components that you saw evident in the facilitator's demonstration of the concept lesson.
- How did the facilitator's use of these components enhance the lesson for the learner?

# Connecting to the Big Idea, Concepts and Skills for Quarter 2

## Fourth Grade Quarterly Concept Organizer

**Number Relationships and Algebraic Reasoning**  
Arithmetic and algebra are guided by properties of operations and equivalence.

**Data Analysis**  
Data can be interpreted from organized visual representations.

Commutative, associative, and distributive properties are inherent in the algorithms for operations of rational numbers.

One of the concepts of algebraic reasoning is balance.

Data can be collected, classified, displayed, and analyzed.

- Add, subtract, multiply, and divide whole numbers.
- Show relationships between operations.
- Solve problems involving addition, subtraction, multiplication, and division.
- Estimate reasonableness.
- Evaluate and use expressions with parentheses.

- Show that equals added or multiplied by equals are equal.

- Create survey questions and collect data.
- Identify mode, median, and outliers.
- Interpret and share data.

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# Rationale

...the depth of students' misunderstandings or the nature of their misconceptions become obvious only when they were asked to explain their thinking...

*Wagner & Parker, 1993*

...unless students are asked to explain their thinking, a teacher may not know which concepts the students understand.

*Manouchebri & Lapp, 2003*

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# About Misconceptions

*Misconceptions* can be defined as "systematic but incorrect rules for accounting for errors in performance."

Errorful rules, then, cannot be avoided in instruction. In fact, they are best regarded as useful diagnostic tools for instructors, who can often use children's systematic errors to detect the nature of children's understanding of a mathematics topic.

Resnick, Nesher, Leonard, Magone, Omanson & Peled, 1989

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# Anticipating Errors and Misconceptions

Review the Fourth Grade task.

- What misconceptions or errors might surface as students work on the task?

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# Scaffolding Student Learning

Appropriate teacher scaffolding of student thinking consists of assisting student thinking by asking thought-provoking questions that preserve the task complexity.

Stein, M., Smith, P., Henningsen, M., & Silver, E., *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development*. Teachers College Press, 2000.

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# Questioning: A Tool for Surfacing Errors and Misconceptions

Review the student responses to the Fourth Grade task. For each response:

- determine what the student knows and understands in terms of the task.
- determine the student's misconception or error.
- determine questions that you would ask to scaffold students' learning without reducing the cognitive demands of the task.

# Addressing Misconceptions and/or Errors

Student Response	What does the student know and understand?	What problem is the student having?	What scaffolding questions might you ask?

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## Questioning: A Tool for Surfacing Errors and Misconceptions in the Concept Lesson

- Review the ways that the concept lesson plan addresses student misconceptions and errors.
- At your table, discuss any additional misconceptions and errors that should be addressed. In what ways could the lesson plan be enhanced to better prepare teachers to anticipate and address student misconceptions and errors?

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# The Role of Student Misconceptions and Questioning

- Why is it important to think about the misconceptions or errors that are likely to surface as students work on a task?
  - What role does questioning play in this process?  
What are the benefits for the student?
  - How can teachers begin to anticipate the difficulties students are likely to have with a particular task?
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# Thinking Through a Lesson Protocol: Content into Practice

- What kinds of planning does this require of teachers?
- With regular use, what pieces of the TTLP could become an integrated part of your practice?