“The only one who truly likes change is a wet baby.”

--Marcia Tate, “Sit and Get” Won’t Grow Dendrites, 2004
Recent studies...have identified important characteristics of highly effective teachers...these studies have found a strong link between teaching practices and knowledge of mathematical content. Although there are many characteristics that distinguish highly effective teachers, one trait emerging from research is the grounding of a teacher’s knowledge of mathematical content, and hence, his or her teaching practices developed around a set of big mathematical ideas.

(Ma 1999, Stigler 2004)
When Big Ideas of Mathematics are understood, mathematics is no longer seen as a set of disconnected concepts, skills, and facts. Mathematics becomes a coherent set of ideas.

Understanding:

- Is motivating
- Promotes more understanding
- Promotes memory
- Influences beliefs
- Promotes the development of autonomous learners
- Enhances transfer
- Reduces the amount that must be remembered

(Lambdin 2003)
Teachers who understand the Big Ideas of mathematics translate that understanding into teaching practices by consistently connecting new ideas to Big Ideas and by reinforcing Big Ideas throughout their teaching.

(Ma, 1999)
Example of a Big Idea in Mathematics

Any number or numerical expression can be represented in equivalent ways.

Examples of Connected Understandings:

- Numbers can be named in equivalent ways (e.g., 2 hundreds 4 tens is equivalent to 24 tens).

- Numerical expressions can be named in equivalent ways (e.g., $1/2 \div 1/4 = 1/2 \times 4/1$; 6 can be thought of as $5 + 1; 10 - 4; 3 \times 2; 42 \div 7$; etc.).

- Every composite number can be expressed as the product of prime numbers in exactly one way, disregarding the order of the factors (Fundamental Theorem of Arithmetic).

- Every fraction/ratio can be represented by an infinite set of equivalent fractions/ratios.
Third Grade Quarterly Concept Organizer

Plane and Solid Shapes
A shape is defined by its attributes, and some attributes can be quantified using measuring tools.

Plane and solid shapes can be classified and analyzed.

An object’s attributes can be measured.

- Identify, describe, and classify plane and solid shapes (polygons and polyhedra).
- Know types of triangles (scalene, obtuse, equilateral, isosceles, right).
- Know attributes of quadrilaterals.
- Identify angles as being right angles or greater than or less than 90º (right, obtuse or acute).
- Construct and deconstruct solid objects.

Place Value, Operations and Equivalence
Numbers are represented in multiple ways and operations are related and are represented in multiple ways.

A comparison of a part to a whole can be represented using fractions.

- Use different tools and units of measurement.
- Find area by using tiles (square units) and volume by using cubes.
- Determine perimeter of polygons.
- Carry out simple unit conversions within a system of measurement.
- Know and use customary and metric unit measurements.

- Compare and order fractions by representing them in drawings or with concrete materials.
- Add and subtract simple fractions.
- Use fractional pieces to represent fractional amounts.
- Know meaning of numerator and denominator.

CA MATH STANDARDS

<table>
<thead>
<tr>
<th>NS 3.1</th>
<th>NS 3.2</th>
<th>AF 1.4</th>
<th>MG 1.1</th>
<th>MG 1.2</th>
<th>MG 1.3</th>
<th>MG 1.4</th>
<th>MG 2.1</th>
<th>MG 2.2</th>
<th>MG 2.3</th>
<th>MG 2.4</th>
<th>MG 2.5</th>
<th>MG 2.6</th>
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</table>
| KEY STANDARDS
| CONCEPT LESSON
| QUARTERLY ASSESSED 3 | 4 | 0 | 1 | 4/CR | 4 | 0 | 4 | 4 | 4 | 0 | 0 | 0 |

DRAFT
“Walk Through the MIG”

Group Sections

- Introduction A, B, & C
- Main Body A
- Main Body B, C, D, & E
- Appendix A
- Appendices B & C
- Appendix D
- Appendix E (I – III)
- Appendix E (IV–VI)
Teaching Mathematics to Create Residue

Los Angeles Unified School District
Elementary Mathematics Department

Mathematics Coaches Professional Development
January 19 – 20, 2006
Setting the Stage

What are your thoughts about the movie clip and about Abbott’s understanding of division?
Objectives

- Define residue
- Identify ways to create residue
What is residue and where would you find it?
“Davis suggested that we have too long been designing our curriculum and instruction on the idea that we should first teach student skills and then have students apply them to solve problems. Davis argued that it is better to begin with problems, allow students to develop methods for solving them, and recognize that what students take away from this experience is what they have learned. Such learning is likely to be deep and lasting. Davis referred to the learning that students take with them from solving problems as “residue.”

-- from Chapter 2 of Making Sense: Teaching and Learning Mathematics with Understanding.
Residue

- Divide participants into groups of 4 or 5
- Each group will begin at 1 of 5 posters.
- Each group will spend 6 minutes at each poster.
  - 3 minutes – read quote, reflect individually
  - 3 minutes – discuss quote in small group
  - Presenter signals when to begin group discussion
  - Use the “Residue Advanced Organizer” to take notes
- Move to next poster
- Repeat until each group has rotated through the 5 posters.
What can you do in your classroom to create the type of thinking that is seen in the “Apollo 13” movie clip?
Closing

- **Solo**: Fill up the “Bathtub Organizer” with elements that create residue.
- **Table**: Share your bathtub residue ideas
- **Whole group**: Share ideas
“Davis suggested that we have too long been designing our curriculum and instruction on the idea that we should first teach students skills and then have students apply them to solve problems. Davis argued that it is better to begin with problems, allow students to develop methods for solving them, and recognize that what students take away from this experience is what they have learned. Such learning is likely to be deep and lasting. Davis referred to the learning that students take with them from solving problems as ‘residue.’” (Pg. 22)

How does this type of learning differ from that seen in the Abbott and Costello video clip?
Poster #1A

“Reflecting means turning something over in your head, thinking again about it, trying to relate it to something else you know. If a task encourages you to reflect on something, you do not rush through it as quickly as you can. Tasks that encourage reflection take time. Communicating means talking and listening.” (Pg. 18)

“For something to be a problem for a student, he or she must see it as a challenge and must want to know the answer. The student must set a goal of resolving the problem. The goal might come from the student, or be adopted by the student after listening to peers or the teacher. The important thing is that the student makes the goal his or her own.” (Pg. 19)

What role do communication, reflection, and goals play in the creation of residue?
Poster #1B
“Tasks that encourage reflection and communication are tasks that link up with students’ thinking. One way to describe this is to say that students should see ways in which they can use the tools they possess to begin the task. We define tools broadly to include things the student already knows and materials that can be used to solve problems. Tools are resources or learning supports…..Using tools to work on mathematical tasks can be thought of like using tools to complete tasks around the home. Tools are very handy and we use many of them without even thinking.” (Pg. 20)

“Tools are used when there is a need to use them, when they can help to solve a problem or complete a task. Tools are used for a purpose. It is likely that you did not practice using a dish cloth just so you could get good at it. There were dirty dishes that needed to be washed. The same is true for mathematical tools. Students get good at using mathematical tools by using them to solve problems. Usually there is little point in practicing with tools just to be practicing.” (Pg. 21)

**Why do tools need to have a purpose?**
**Give a specific example of using a tool to create residue.**

Duster slippers' for cats: now the most boring job around the house becomes hours of fun. (See below for more information."

Chindogu is the Japanese art of inventing ingenious everyday gadgets that, on the face of it, seem like an ideal solution to a particular problem. However, Chindogu has a distinctive feature: anyone actually attempting to use one of these inventions, would find that it causes so many new problems, or such significant social embarrassment, that effectively it has no utility whatsoever. Thus, Chindogu are sometimes described as 'unuseless' - that is, they cannot be regarded as 'useless' in an absolute sense, since they do actually solve a problem; however, in practical terms, they cannot positively be called 'useful'. For more information, go to http://en.wikipedia.org/wiki/Chindogu
Poster #1C
“Thinking of understandings as outcomes of solving problems rather than as concepts that we teach directly requires a fundamental change in our perceptions of teaching. Many of us have been brought up to think that the best way to teach mathematics is to teach important concepts, like place value or common denominators, by explaining them clearly and demonstrating how to use them and then having students practice them. Our recommendation is that we change our way of thinking and teaching so that students are allowed to develop concepts, such as place value and common denominators, in the context of solving problems. This means that when selecting tasks or problems, we need to think ahead about the kinds of relationships that students might take with them from the experience.” (Pg. 22)
Summarize these ideas into a simple sentence and share with your group.
Poster #1D
“[There are] two types of residue that are essential and that can provide useful guides for selecting tasks. One type can be called insights into the structure of mathematics, and the second type is the strategies or methods for solving problems.”  (Pg. 23)

“Tasks that encourage students to reflect on mathematical relationships are likely to leave behind insights into structure [i.e. residue]. If tasks are problematic for students and if students are allowed to work out methods to complete the tasks, then they also are likely to take with them strategies for solving problems.”  (Pg. 23)

“Two kinds of strategies will be left as residue. One kind of strategy is a specific technique for completing specific kinds of tasks…. [the second type of strategy is students’ developing] general approaches for inventing specific procedures or adapting ones they already know to fit new problems.  (Pg. 23-24)

**Explain in your own words the 2 types of residue and create an example of each.**
Poster #1E
“A major advantage of thinking about learning as the residue that gets left behind when solving problems is that it provides a way of dealing with a very common difficulty. Many students have trouble connecting the concepts they are learning with the procedures they are practicing.” (Pg. 24)

“[Students] learn procedures by imitating and practicing rather than by understanding them, and it is hard to go back and try to understand a procedure after you have practiced it many times.” (Pg. 25)

“If students are encouraged to develop their own procedures for solving problems, then they must use what they already know, including the understandings they have already constructed. There is no other way to do it. Understandings and procedure remain tightly connected because procedures are built on understandings. The methods students first develop may not be the most efficient ones, but they will be methods students understand.” (Pg. 25)

“Much of the content in current curricula, as presented in popular textbooks, is appropriate as long as students are allowed to make the mathematics problematic… Tasks should be selected for the mathematics of the situation, rather than other extraneous features and that, as [students] complete the task and look back, the mathematics of the situation should be the most salient residue. (Pg. 26)

How does having students develop their own mathematical procedures promote residue?
## Residue Advanced Organizer

<table>
<thead>
<tr>
<th>Poster</th>
<th>Question</th>
<th>Notes</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>What role do communication, reflection, and goals play in the creation of residue?</td>
<td></td>
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<tr>
<td>2</td>
<td>Why do tools need to have a purpose? Give a specific example of using a tool to create residue.</td>
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<tr>
<td>3</td>
<td>Summarize these ideas into a simple sentence.</td>
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<tr>
<td>4</td>
<td>Explain in your own words the 2 types of residue and create an example of each.</td>
<td></td>
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<tr>
<td>5</td>
<td>In the long run, why is a student’s own method more important than using the traditional procedures?</td>
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</tbody>
</table>
What creates residue for your students?
<table>
<thead>
<tr>
<th>A. Most Practical</th>
<th>B. Most Thought Provoking</th>
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<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td>C. Most Promising for</td>
<td>D. Biggest Surprise</td>
</tr>
<tr>
<td>Student Achievement</td>
<td></td>
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</tbody>
</table>
RIGOR DEFINED

Session Objectives:

• Engage in a 4th grade additional lesson and a textbook lesson as a learner.

• Evaluate the activities based on the level of thinking that students would be engaged in.
RIGOR DEFINED

Agenda:
• Opening: Morning Reflection
• Paint the Building (Additional Lesson)
• Find a Rule (Textbook Lesson)
• Task Analysis Guide
• Task Sort
• Closing: Rigor Defined Similes
“Not all tasks are created equal, and different tasks will provoke different levels and kinds of student thinking.”
- Stein, Smith, Hennington, & Silver, 2000
Tasks form the basis for students’ opportunities to learn what mathematics is and how one does it, yet not all tasks afford the same levels and opportunities for student thinking.
PAINT THE BUILDING

• On Monday, a painter had to paint a building that was shaped like a cube. When she read the label on the can of paint, she realized one can of paint would cover one face of the building. She had to paint all four sides and the roof of the building.

• On Tuesday, she had to paint the building next door. It was the size of two of the first buildings put together.

• On Wednesday, she had to paint the third building on the block. It was the size of three of the cubic buildings put together.

• On Thursday, she had to paint yet another building that was, of course, like four of the cubic buildings put together.
Your Job:

• Figure out how many gallons of paint she would need for each day, Monday through Thursday.

• On a T-chart, continue this pattern up to a building 10 cubic units long.

• Create a formula to help you figure out how many gallons of paint it would take to paint a building 23 cubic units long.
“Davis suggests that we have too long been designing our curriculum and instruction on the idea that we should first teach students skills and then have students apply them to solve problems. Davis argued that it’s better to begin with problems, allow students to develop methods for solving them, and recognize that what students take away from this experience is what they have learned.”
“There is no decision that teachers make that has a greater impact on students’ opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.”

-Lappan & Briars, 1995
“By analyzing two tasks that are mathematically similar and categorizing a set of tasks, we can begin to differentiate between tasks that require thinking and reasoning and those that require the application of previously learned rules and procedures.”
“The level and kind of thinking which students engage determines what they will learn.”
- Hiebert, Carpenter, Fennema, Fuson, Weame, Murray, Oliver, & Human, 1997
Goals for Sorting Task Activity:

• To raise awareness of how mathematical tasks differ with respect to their cognitive demands

• To highlight the importance of analyzing and discussing tasks in order to determine the level of thinking required to solve them (superficial features can trick you!!)
Characterizing Mathematical Tasks in Terms of Cognitive Demands

<table>
<thead>
<tr>
<th>Task</th>
<th>Low Level</th>
<th>High Level</th>
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<tbody>
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<td>A</td>
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“If we want students to develop the capacity to think, reason, and problem solve then we need to start with high-level, cognitively complex tasks.”

–Stein and Lane, 1996
“A rigorous lesson is like...”
A rigorous lesson is like a group therapy session; when you talk about your problem, you get a clearer understanding of your outlook from the feedback you receive.
A rigorous lesson is like a Thomas Guide; there are many paths that lead to the same destination.
Complete These Statements

The ________________

is organized around big ideas, concepts and skills.
A ________ is a statement of an idea that is central to the learning of mathematics, and is an idea that links numerous mathematical understandings into a coherent whole.
________________ will be incorporated throughout the mathematics instructional roadmap, one per quarter, and will be used to help students develop understanding around key concepts and big ideas.
Some tasks involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory.
The four kinds of tasks as categorized by the Task Analysis Guide are memorization, procedures without connections, procedures with connections and __________________.
Not all _______ are created equal, and different __________ will provoke different levels and kinds of student thinking.
The concept lesson includes a set up phase, an __________ phase and a share, discuss and analyze phase.
The only one who truly likes change is a __________.
When Big Ideas of Mathematics are understood, mathematics is no longer seen as a set of ______ concepts, skills, and facts. Mathematics becomes a ______ set of ideas.
Procedures ________
connections tasks
require some degree of
cognitive effort.
_____________ is the learning that students take away with them after they solve problems.
For something to be a problem for a student, he or she must see it as a ______ and must want to know the __________________.